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XX.—ACOUSTICAL EXPERIMENTS WITH A MECHANICAL VIBRATOR.

By E. MALLETT, *D.Sc., M.I.E.E., A.M.I.C.E.*

Received December 15, 1926.

ABSTRACT.

In this Paper some preliminary experiments are described with a mechanical device vibrating a piston at one end of a tube so that a sound wave is emitted at the other. The particle velocities in the sound wave are measured by a Rayleigh disc, and resonance curves are drawn. It is shown how the energy in the sound wave can be calculated from the results obtained. The experiments are directed towards obtaining a standard source of sound, and the results are encouraging, but a considerable amount of work remains to be done.

CONTENTS.

- I. Description of apparatus.
- II. Sound amplitude measurements :—
 - (i) Measurement of amplitude of piston with different rollers ;
 - (ii) Curves of sound amplitude for different piston amplitudes ;
 - (iii) Resonance curves.
- III. Pipe theory on electrical analogy.
- IV. Numerical results and conclusions.
- Appendix: Critique of Paper on "The Degradation of Acoustic Energy."

I. DESCRIPTION OF APPARATUS.

A MECHANICAL vibrator was used to give known amplitudes of vibration of variable frequency to a piston at the end of a pipe, so as to produce the simplest condition possible from the mathematical point of view. The principle on which the vibrator works will be understood from Fig. 1. *R* is a roller, with an eccentric spindle at *B*, and resting on a large driving wheel *D*. The spindle of the roller is carried at the end of a rocking arm *AB*, pivoted at *A*, and there is a means provided of keeping the surfaces of *R* and *D* in contact. When *D* is rotated *R* will be rotated, because of the friction between *R* and *D*, and *B* will execute a vibratory movement along a vertical line, because it is not centrally placed in *R*. Further,

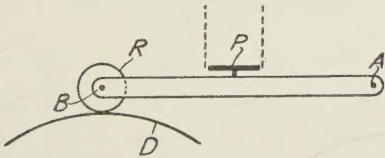


FIG. 1.

since the spindle is rigidly fixed in the arm *BA*, the whole arm will be caused to rock about the pivot *A*, and the piston *P* fitting in the end of a pipe will also vibrate along the axis of the pipe.

Naturally such an apparatus was not made without many difficulties arising. The means of holding the roller on to the driving wheel, for instance, was a difficulty finally solved by using a thick rubber block pressing upon the end of the arm. Lubri-

cation of the roller was effected by means of a wick passing through a hole in the centre of the spindle with radial cuts to the bearing surface. Mechanical resonances in the bar and in the framework holding the whole contrivance occurred, and were investigated by fixing an iron armature to the arm and a telephone receiver to the frame with its poles close to the armature, so that as the arm vibrated an electromotive force was produced in the receiver coils.

The difficulties and the way in which they were overcome, with a complete description of the final apparatus are given in the writer's M.Sc. thesis in the University of London Library. This apparatus gives practically a sinusoidal vibration free from resonances, that is of constant amplitude, up to a frequency of about 500 cycles per second. The actual vibration of the spindle was shown to be expressed by

$$x = k \left(\cos \omega t - \frac{1}{2} \frac{k}{a} \cos 2 \omega t \right) \dots \dots \dots (1)$$

where k is the eccentricity and a the radius of the roller. Higher harmonics than the second are quite negligible.

To convert the vibrator to a mechanically-driven organ pipe the whole

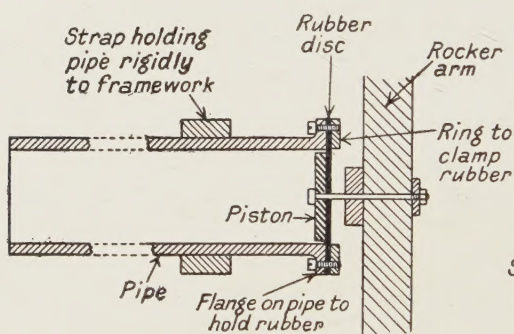


FIG. 2.

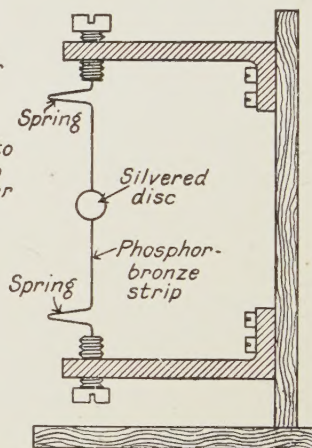


FIG. 3.

apparatus was turned through a right angle, so that the piston P became vertical and the pipe horizontal. The piston was made as good a fit in the pipe as possible, and the end of the pipe was made airtight by means of a rubber disc fixed as shown in Fig. 2. The pipe was rigidly clamped to the frame, and the far end was also supported.

It was the original intention to obtain different amplitudes of vibration by fixing the apparatus to be vibrated at different positions along the rocker arm; but with the present pipe arrangement this proved to be impracticable, so a set of rollers was made having the same diameter but different eccentricities.

The driving wheel was driven by an electric motor, and changes of frequency were obtained by simply altering the motor speed.

Measurements of the particle velocities of the sound wave from the pipe were made by means of a Rayleigh disc arranged as shown in Fig. 3. This robust arrangement was possible, as the sound velocities to be measured were large.

II. SOUND AMPLITUDE MEASUREMENTS.

(i) *Amplitudes of Piston with Different Rollers.*

Four rollers were made each of $\frac{1}{4}$ in. diameter, but having different eccentricities, and so giving different amplitudes of vibration to the piston. The amplitudes were measured by a microscope while the machine was at rest by rotating the driving wheel and taking the difference of the readings with the vibrating bar in its top and bottom positions. The measurements were made at a distance of 12.15 in. from the pivot, the distance of the piston was 8 in. from the pivot, and that of the roller 16 in. from the pivot. The actual diameters of the rollers were measured by a micrometer gauge. Details of the rollers are given in the following table :—

Roller number.	Diameter of roller.	Measured lift of bar.	Lift of roller.	Throw of piston.	Amplitude of piston.
	Ins.	Ins.	Ins.	Ins.	Ins.
1	0.2496	0.00600	0.00790	0.00395	0.00197
2	0.2496	0.00380	0.00500	0.00250	0.00125
3	0.2494	0.00295	0.00388	0.00194	0.00097
4	0.2498	0.00074	0.000975	0.00048	0.00024

(ii) *Sound Amplitude with Different Rollers.*

The Rayleigh disc was now placed in front of the organ pipe and the machine speeded up. Two marked resonances were obtained within the allowable speed limit, one corresponding to the fundamental mode of the pipe, and the other to its first overtone, at three times the frequency. The maximum deflections of the disc were measured at these two frequencies with each of the four rollers in turn, and a figure proportional to the particle velocity obtained as the square root of the deflection.

The readings were as follows :—

Roller number.	Piston amplitude. Ins.	Fundamental.		First overtone.	
		Deflection of disc. δ	$\sqrt{\delta}$	Deflection of disc. δ	$\sqrt{\delta}$
1	0.00197	5.6	2.36	28.0	5.30
2	0.00125	2.0	1.41	10.3	3.21
3	0.00097	1.3	1.14	6.4	2.51
4	0.00024	—	—	0.4	0.63

The sound amplitudes ($\sqrt{\delta}$) are plotted against the piston amplitudes in Fig. 4. It is seen that two straight lines result, each passing through the origin. This quite satisfactorily indicates that the particle velocity in the sound wave outside the tube is directly proportional to the piston amplitude.

(iii) *Resonance Curves.*

Resonance curves taken with different rollers by varying the speed of the motor, and therefore the frequency of vibration of the piston, and measuring the particle velocity at the mouth of the tube by means of the Rayleigh disc are shown in Fig. 5 for the fundamental vibration, and in Fig. 6 for the first overtone.

The speed of the driving wheel was measured by a Moule tacheometer fixed to its spindle, and the frequency of vibration calculated on the assumption of no slip between wheel and roller, which has been shown to be at any rate very nearly true. No importance can, however, be attached to the apparent variations of resonant frequency with amplitude indicated by the curves. The frequency determinations were not sufficiently accurate for this, as the Moule tacheometer seemed to vary a little from day to day, and the temperature was not always quite the same. But it is thought that the differences of frequency are fairly accurate for any one curve.

Another difficulty was to keep the speed of the motor sufficiently constant for accurate readings both of the Rayleigh disc deflection and of the frequency to be

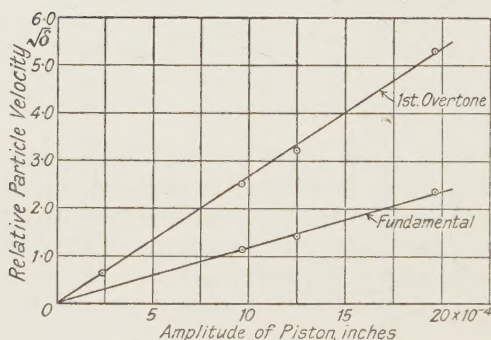


FIG. 4.

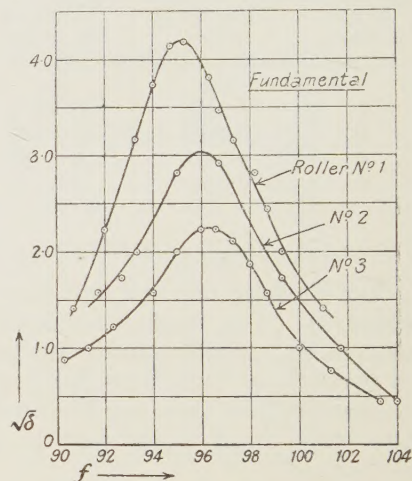


FIG. 5.

taken. The method adopted was to brake the driving wheel judiciously with the finger to keep the Rayleigh disc deflection steady, and then read the tacheometer. Then, again, to avoid room resonances, such curves as these should be taken in a sound absorbing chamber, but none was available at the time. The presence of "coupled circuit" distortion is indicated in the fundamental resonance curves. These curves can therefore only be looked upon as preliminary. An efficient speed control, an accurate method of frequency measurement, and an absorbing chamber are necessary for more exact work.

III. PIPE THEORY ON ELECTRICAL ANALOGY.

The theory of the acoustical vibrations in the pipe can be written down by analogy with the electrical oscillations in a transmission or telephone line, particle velocities in the acoustical case corresponding with electrical currents, and alter-

nating air pressures with electrical potential differences. In the previous experiments the particle velocity at the "sending end" of the pipe is that of the piston, while the Rayleigh disc measurements give a figure which is proportional to the particle velocity at the "receiving end"—that is at the open end of the pipe. Thus the telephone equation likely to be of most value is

$$\frac{I_s}{I_r} = \cosh Pl + \frac{Z_r}{Z_0} \sinh Pl \quad (2)$$

where I_s = sending end particle velocity = piston velocity
 I_r = receiving end velocity
 l = length of pipe

and $P = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$

is the propagation constant of the pipe line. Numerous experiments have shown that the loss of energy in a smooth tube such as the one used is very small. For

instance, in an experiment in which resonance curves were obtained by altering the pipe length (see Acoustic Experiments with Telephone Receivers, J.I.E.E., 63, p. 512, Fig. 16) the resonance curve at the second overtone of the pipe was substantially the same as that of the fundamental, although the pipe length had been increased from about 7 cms. to about 42 cms. The R and G of the pipe can therefore be neglected, and the propagation constant is

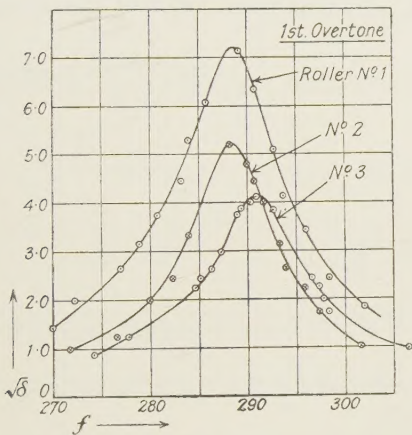


FIG. 6.

$$P = j\omega\sqrt{LC} = j\beta.$$

L , in the acoustical case, is m , the linear mass in grams per cm. run, which is equal to ρS , where ρ = density of air, and S = cross-sectional area of pipe.

$\frac{1}{C}$ is the "stiffness" of the air, $s = S\gamma\rho_0$ dynes, where ρ_0 is the normal pressure of the air, and γ the ratio of the specific heats.

The wave velocity (velocity of sound)

$$= \frac{1}{\sqrt{LC}} = \sqrt{\frac{S}{m}} = \sqrt{\frac{\gamma\rho_0}{\rho}} = c \text{ cms./sec.}, \beta = \omega\sqrt{LC} = \frac{\omega}{c} \text{ cm}^{-1}$$

$$Z_0 \text{ the characteristic impedance} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} = mc \quad \frac{\text{grams.}}{\text{sec.}}$$

Z_r is the load impedance, occasioned in the acoustical case by the pipe being open to the air at the end. Z_r may be written as $R + jX$, and both R and X are unknown. The dimensions of R and X must be gms./sec.

Returning the electrical symbols as they are more familiar, the ratio of the particle velocities at the two ends of the tube becomes

$$\begin{aligned}\frac{I_s}{I_r} &= \cosh j\omega\sqrt{LC}l + (R+jX) \sqrt{\frac{C}{L}} \sinh j\omega\sqrt{LC}l \\ &= \sqrt{1+X^2\frac{C}{L}} \cos \left\{ \omega\sqrt{LC}l + \varphi \right\} + jR \sqrt{\frac{C}{L}} \sin (\omega\sqrt{LC}l),\end{aligned}$$

where $\tan \varphi = X \sqrt{\frac{C}{L}} \dots \dots \dots (3)$

The locus of this expression as ω varies on the assumption that R and X remain constant, and neglecting φ for the moment, is an ellipse, with, in the present case, the horizontal major axis $\sqrt{1+X^2\frac{C}{L}}$ much greater than the vertical minor axis, $R\sqrt{\frac{C}{L}}$. The minimum value of the ratio—i.e., the maximum value of I_r —occurs then (still neglecting φ), when $\cos \omega\sqrt{LC}l = 0$ —i.e., when $\omega\sqrt{LC}l = \frac{2m+1}{2}\pi$, where m is any integer, including 0.

This is the same as $\frac{\omega}{c}l = \frac{2m+1}{2}\pi$,

or, $f = \frac{2m+1}{4} \cdot \frac{c}{l}$,

the familiar expression for the resonant modes of a pipe open at one end and closed at the other.

Taking into account the modification introduced by the terminal impedance as φ , for the fundamental resonance occurs when

$$\omega\sqrt{LC}l + \varphi = \frac{\pi}{2},$$

$$\omega = \left(\frac{\pi}{2} - \varphi \right) \cdot \frac{1}{\sqrt{LC}l},$$

or, $f = \frac{C}{4l} - \frac{\varphi}{2\pi l} \cdot c$.

Thus the resonant frequency is lowered. This is usually taken account of by assuming that the pipe is in effect a little longer—the well known “end correction.”

φ is small, so that approximately

$$\varphi = \tan \varphi = X \sqrt{\frac{C}{L}},$$

and

$$f = \frac{c}{4l} - \frac{1}{2\pi} \cdot \frac{X}{Ll}$$

Proceeding to the first overtone, resonance occurs when

$$\omega\sqrt{LCl} + \phi^1 = \frac{3\pi}{2}.$$

leading to

$$f = \frac{3}{4} \cdot \frac{c}{l} - \frac{\phi'}{2\pi\sqrt{LCl}}.$$

In order that this may be three times the frequency of the fundamental, ϕ' must be three times ϕ —i.e.,

$$\phi' = \tan \phi' = 3X\sqrt{\frac{C}{L}}.$$

Similarly for the higher modes, the reactance of the load must increase, in order to fit in with the classical experimental observations. Writing $X = \omega L'$, this is the same as saying that L' remains constant.

For then resonance occurs when

$$\omega\sqrt{LCl} + \omega L' \sqrt{\frac{C}{L}} = \frac{2m+1}{2} \pi,$$

or,

$$f = \frac{2m+1}{4} \cdot \frac{c}{\left(l + \frac{L'}{L}\right)} \quad \dots \dots \dots (4)$$

$\frac{L'}{L}$, therefore, is the end correction, and L' can be determined experimentally by accurate observations of the resonant frequencies.

The load resistance R is found from the resonance curves. In equation (3) $X^2 \frac{C}{L} = \frac{\omega^2 (L')^2}{c^2} \cdot \frac{L'}{L}$ will be something less than 1 cm., and $\frac{\omega^2}{c^2}$ is very much less than one, so that $\sqrt{1 + X^2 \frac{C}{L}}$ is very nearly one. Also, since ϕ is small, at or near the resonances, when $\cos(\omega\sqrt{LCl} + \phi) = 0$, $\sin(\omega\sqrt{LCl})$ will be very nearly one, positive or negative according to the overtone. Considering small departures of ω from the various resonant values ω_0 .

$$\begin{aligned} \cos\{\omega\sqrt{LCl} + \phi\} &= \cos\{(\omega - \omega_0)\sqrt{LCl} + \omega_0\sqrt{LCl} + \phi\}, \\ &= \cos\left\{(\omega - \omega_0)\sqrt{LCl} + \frac{2m+1}{2}\pi\right\}, \\ &= (\omega - \omega_0)\sqrt{LCl} \times (-1)^m \end{aligned}$$

very nearly, and $\sin(\omega\sqrt{LCl}) = (-1)^m$ very nearly.

So that equation (3) can be written

$$\frac{I_s}{I_r} = (-1)^m \left\{ (\omega_0 - \omega)\sqrt{LCl} + j\sqrt{\frac{C}{L}} R \right\} \quad \dots \dots \dots (5)$$

The curves have been examined according to this expression in two ways (a) by considering only the sizes of the quantities, and (b) by the circle construction.

(a) Considering sizes only, equation (5) gives

$$\begin{aligned}\left(\frac{I_s}{I_r}\right)^2 &= (\omega_0 - \omega)^2 LC l^2 + \frac{C}{L} R^2, \\ &= 4\pi^2 LC l^2 (f_0 - f)^2 + \frac{C}{L} R^2.\end{aligned}$$

Since I_s is very nearly constant over the resonance, and the readings δ of the

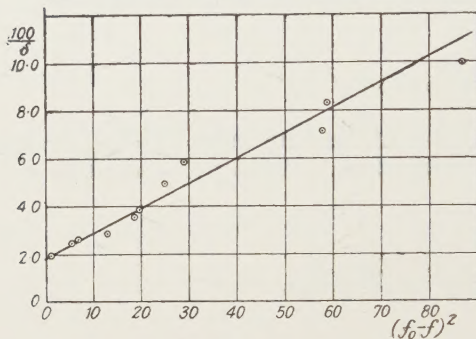


FIG. 7.

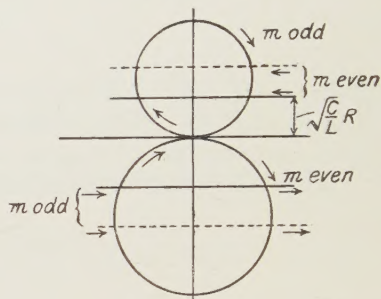


FIG. 8.

Rayleigh disc are proportional to I_r^2 , say, $I_r^2 = A\delta$

$$\frac{1}{\delta} = \frac{A}{I_s^2} \cdot 4\pi^2 LC l^2 (f_0 - f)^2 + \frac{A}{I_s^2} \cdot \frac{C}{L} R^2.$$

So that plotting $\frac{1}{\delta}$ against $(f_0 - f)^2$ a straight line should be obtained whose slope is $\frac{A}{I_s^2} \cdot 4\pi^2 LC l^2$, and whose intercept $= \frac{A}{I_s^2} \cdot \frac{C}{L} R^2$, and the ratio intercept/slope $= \frac{R^2}{4\pi^2 L^2 l^2}$

and

$$\frac{R}{Ll} = 2\pi \sqrt{\frac{\text{Intercept}}{\text{Slope}}}$$

The curves of Figs. 5 and 6 were all examined in this way. As an example, the figures for the first overtone with No. 1 roller are given in the following table, and the curve $\frac{100}{\delta}$ plotted against $(f_0 - f)^2$ is drawn in Fig. 7.

No. 1 Roller.—Resonant frequency = 288.3 cycle/sec.

f	$f_0 - f$	δ	$(f_0 - f)^2$	$\frac{100}{\delta}$
279	9.3	10	87	10
280.7	7.6	14	58	7.15
283.3	5.0	20	25	5.0
284	4.3	28	18.5	3.6
289.3	1.0	51	1.0	2.0
290.7	-2.4	40	5.7	2.5
292.7	-4.4	26	19.4	3.84
293.7	-5.4	17	29	5.9
296	-7.7	12	59	8.3
285.7	2.6	37	6.7	2.7
284.7	3.6	35	13	2.86

In this case the slope of the line is 0.105 and the intercept is 1.8. So that

$$\frac{R}{Ll} = 2\pi \sqrt{\frac{1.8}{.105}} = 26.1.$$

The results with the other rollers form similar curves, and are collected in the following table :—

Roller Number.	Fundamental.			First overtone.		
	Intercept.	Slope.	$\frac{R}{Ll}$	Intercept.	Slope.	$\frac{R}{Ll}$
1	5.3	1.17	13.4	1.8	0.105	26.0
2	11.0	1.7	16.0	3.5	0.295	21.6
3	19.5	3.1	15.7	6.0	0.35	26.0
			Mean = 15			
						Mean = 25

The differences of the values of $\frac{R}{Ll}$ found with different rollers can only be attributed to experimental error at this stage. There is no indication of any increase of load with amplitude.

(b) The examination of the curves by the circle and straight line method* is perhaps simpler and more illuminating. Equation (5) gives

$$\frac{I_r}{I_s} = \frac{1}{(-1)^m \left\{ (\omega_0 - \omega) \sqrt{LCl} + j \sqrt{\frac{C}{L}} R \right\}}$$

The locus of the denominator over the resonant ranges of frequency is a series of horizontal straight lines, above and below the origin in order of the harmonic.

These inverted to give $\frac{I_r}{I_s}$ become a series of circles, as indicated in Fig. 8, below the origin with m even and above the origin with m odd, the circle described clockwise in each case as ω is increased. The phase angle α between I_r and I_s is evidently given by

$$\tan \alpha = \frac{(\omega_0 - \omega) \sqrt{LCl}}{\sqrt{\frac{C}{L}} R} = (\omega_0 - \omega) \frac{Ll}{R}$$

so that the circle construction carried out on the resonance curve gives for the resulting straight line, plotting ω ,

$$\frac{d\omega}{d \tan \alpha} = \frac{R}{Ll}$$

or, plotting frequencies, as has been done,

$$\begin{aligned} \frac{R}{Ll} &= 2\pi \cdot \frac{df}{d \tan \alpha}, \\ &= 2\pi \times \text{slope of line.} \end{aligned}$$

The construction is carried out in Fig. 9 on the first overtone with No. 2 roller, and gives quite a satisfactory straight line with a slope of 3.39. This multiplied by $2\pi = 21.4$ in close agreement with the value for $\frac{R}{Ll}$ found by the other method.

* J.I.E.E., Vol. 62, p. 524.

IV. NUMERICAL RESULTS AND CONCLUSIONS.

It is proposed in this section to make calculations of the acoustical energies

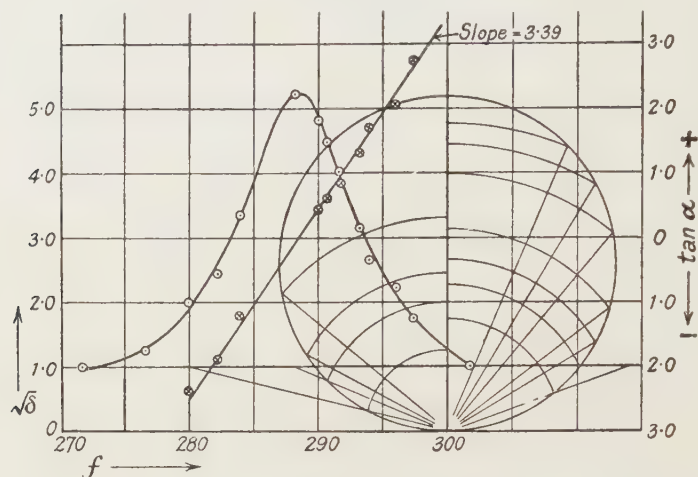


FIG. 9.

involved and to indicate some directions in which the work may usefully be extended. The following details are required:—

Length of pipe = 89 cms. = l .

Internal diameter of pipe = 1.98 cms.

Area of pipe = $S = 3.1$ sq. cms.

Velocity of sound (from Kaye and Laby) at $22^\circ\text{C}.$, the temperature at

which the resonance curves were taken = 34,440 cms./sec. = c .

At $22^\circ\text{C}.$, fundamental resonance frequency without end correction = $\frac{c}{4l} = 96.8$ cycles per second.

The actual frequency from the resonance curves was about 96.0, corresponding to a pipe length of 89.8 cms., or an end correction of 0.8 cm. However, Kaye and Laby give end correction with no flange of $0.57 \times \text{radius} = 0.57$ cm. The frequency measurements were not sufficiently accurate for this determination, so the end correction will be taken as 0.6 cm. Thus,

$$\frac{L'}{L} = 0.6 \text{ cm.}$$

$$L = m = \rho S = 0.001196 \times 3.1 \text{ gms./cm.}$$

$$= 0.0037 \text{ gms./cm.} = m.$$

$$L' = 0.0022 \text{ grams.}$$

	Fundamental.	First overtone.
$\frac{R}{L}$	15	25 $\frac{1}{\text{sec.}}$
R	$= 15 \times 0.0037 \times 89$	$25 \times 0.0037 \times 89$
	$= 5.0$	$= 8.3 \text{ gms./sec.}$
$X = \omega L'$	$2\pi \times 96 \times 0.0022$	$2\pi \times 288 \times 0.0022$
	$= 1.33$	$= 4.0 \text{ gms./sec.}$
$Z_r = R + jX$	$5.0 + j1.33$	$8.3 + j4.0 \text{ gms./sec.}$
$\tan \phi = X \sqrt{\frac{C}{L}} = \omega \left(\frac{L'}{L} \right)$	$\frac{2\pi \times 96}{34,440} \times 0.6 = 0.0105$	$\frac{2\pi \times 288}{34,440} \times 0.6 = 0.0316$
ϕ	0.6°	1.8°

The acoustical energy radiated per second is $I_r^2 R$ (on the assumption, of course, that no energy is lost in the tube), where I_r is the root-mean-square value of the particle velocity in the mouth of the tube. At resonance this is from equation (5) given by

$$\frac{I_s}{I_r} = \sqrt{\frac{C}{L}} R = \frac{R}{mc},$$

so that

$$I_r = I_s \times \frac{mc}{R}.$$

Hence, power radiated

$$W = I_s^2 \frac{m^2 c^2}{R} \text{ ergs/sec.} = \frac{(\omega a)^2 m^2 c^2}{2R} \times 10^{-7} \text{ watts}$$

where a is the piston amplitude.

For the various rollers the particle velocities and energies are collected below : —

Roller number.	Amplitude a cms.	Fundamental.			First overtone.		
		$f=96$			$f=288$		
		I_s	I	W	I_s	I_r	W
		(max.) cms. sec.	(max.) cms. sec.	(milli-watts)	(max.) cms. sec.	(max.) cms. sec.	(milli-watts)
1	0.0050	3.0	76	1.4	9.0	139	8.0
2	0.00318	1.9	48	0.58	5.7	88	3.2
3	0.00247	1.5	38	0.36	4.5	69	2.0

The Rayleigh disc was calibrated in a steady air stream, with a result that the max. value of the alternating particle velocity should be given by $10\sqrt{\delta}$. Thus the particle velocities measured at resonance with the No. 1 roller were 42 cms./sec. for the fundamental and 72 cms./sec. for the overtone, whereas the values of I_r were 76 and 139 cms./sec. respectively. Thus, while these figures are of the same order, it is evident that the Rayleigh disc is not measuring I_r . In fact, it would hardly be expected to do so, as the disc was not wholly in the mouth of the pipe, but projected from the end, and the sound intensities are falling off very rapidly as the wave spreads out and becomes spherical.

The figures obtained do, however, provide a demonstration that the calibration of the disc is the same at 96 cycles per second as at 288 cycles per second. The particle velocity V in a spherical wave at a distance r from the source is given by

$$V_r = \frac{A}{4\pi} \sqrt{\frac{1}{r^2} \cdot \frac{\omega^2}{c^2} + \frac{1}{r^4}},$$

and $\frac{\omega^2}{c^2}$ is very much less than one and r of the order 1 cm. So that very nearly

$V_r = \frac{A}{4\pi} \cdot \frac{1}{r^2}$; that is, the measurements made will be independent of the frequency

but depend only on the factor A , the amplitude of the source. At resonance for the fundamental

$$\frac{I_{r_0}}{I_{s_0}} = \frac{mc}{R_0} = \frac{0.0037 \times 34,400}{5} = 25.4,$$

and for the first overtone

$$\frac{I_{r_1}}{I_{s_1}} = \frac{mc}{R_1} = 15.4.$$

The ratio of the velocities V_{r_0} and V_{r_1} for the fundamental and the first harmonic at resonance should, therefore, if the calibration of the Rayleigh disc is the same at each frequency, be given by

$$\frac{V_{r_1}}{V_{r_0}} = \frac{I_{r_1}}{I_{r_0}} = \left(\frac{I_{s_1}}{I_{s_0}} \right) \times \frac{15.4}{25.4} = 3 \times \frac{15.4}{25.4} = 1.8,$$

since the ratio of I_{s_1} to I_{s_0} is equal to the frequency ratio as the amplitude is the same.

The maximum values of $\sqrt{\delta}$ for the different rollers are given as follows from the resonance curves:—

Roller number.	Maximum $\sqrt{\delta}$		Ratio.
	First overtone.	Fundamental.	
1	7.2	4.4	1.64
2	5.2	3.05	1.7
3	4.15	2.25	1.85
Mean = 1.73			

The mean value of the ratio 1.73 is in quite good agreement with the value 1.80 found above.

The apparatus described has been shown to be capable of giving sound waves of variable frequency and amplitude which can be reproduced at will. It constitutes, in fact, a "standard source of sound." The actual frequencies obtained at present are low, but it is hoped by a modification of the design to obtain far higher frequencies. When a sound absorbing chamber is available it is hoped to extend the investigation to find, if possible, the actual loss in the tube, at the piston especially, so that the total power W radiated can be estimated with accuracy. With this knowledge the value A of the equivalent point source can be found from the expression

$$W = \frac{A^2 \rho \omega^2}{8\pi c}.$$

Then by making velocity measurements with a Rayleigh disc in front of the tube at various distances when

$$V_r = k\sqrt{\delta} = \frac{A}{4\pi} \sqrt{\frac{1}{r^2} \cdot \frac{\omega^2}{c^2} + \frac{1}{r^4}}$$

the value of k can be found. In other words, the possibility arises of calibrating the Rayleigh disc absolutely at the frequency at which it is to be used.

The work described has been spread out over a number of years. The vibrator was made seven years ago in Prof. J. T. MacGregor-Morriss's laboratories at East London College, where Mr. Weaire and Mr. Niblett gave assistance in its construction. The adaption to the present acoustical work was carried out by Mr. Andrews at the City and Guilds (Eng.) College four years ago, and assistance in the actual measurements was given by Mr. F. S. Smith, a senior student. To Prof. MacGregor-Morris, in the early stages, and to Prof. C. L. Fortescue later the author has been indebted for ever-ready discussion and suggestion.

APPENDIX.

Critique of Mr. Hart's Paper on "The Degradation of Acoustical Energy" (Proc. Roy. Soc., A, Vol. 105, p. 80, 1924).

Mr. Hart in his Paper, which is reproduced in a book, "The Principles of Sound Signalling," by M. D. Hart and W. W. Smith, claims to show that acoustical energy propagated through air suffers a loss in propagation, the loss being greater the greater the flux of energy. This "degradation" is expressed mathematically by $f\Omega_E \frac{1}{cm}$, the

figure by which E must be multiplied to obtain the loss per period from unit volume of the medium at sound intensity E (ergs per sq. cm. per second) and frequency f . The actual loss per second per c.c. is thus $f_f\Omega_E E$. From experiments carried out at a frequency of 100 cycles per second values of $f\Omega_E$ are deduced, rising to 3.0 at $E=18 \times 10^{-4}$ ergs per sq. cm. per sec.

In the pipe experiments described above the sound intensities at 96 cycles reached 1.4 milliwatts=14,000 ergs per sec., and since the cross-sectional area of the pipe was 3.1 sq. cm., the flux of energy in the pipe was 4,500 ergs per sq. cm. per sec., a figure enormously greater than Mr. Hart's maximum at about the same frequencies, and which should involve, presumably, a far greater value of Ω . But taking only the value of 3.0 corresponding to $E=18 \times 10^{-4}$, the energy loss would be $3 \times 4,500$ ergs per c.c. per period, and the loss per second would be 96 times this, or 1.3×10^6 ergs per c.c. This energy loss taking place in the air itself would, presumably, heat the air, the amount of heat produced being $\frac{1.3 \times 10^6}{4.2 \times 10^7} = 0.03$ calories per c.c. per sec.

Assuming no loss by radiation, the temperature rise every second would then be, taking the specific heat of air as 0.2417 calories per gram, or $0.2417 \times 0.0012 = 2.9 \times 10^{-4}$ calories per c.c., $0.03/2.9 \times 10^{-4} = 103^\circ\text{C}$ —an absurdity. Actually, a sensitive thermo-couple was inserted in the tube, but no rise of temperature whatever could be detected.

It is clear also that if any appreciable loss were occurring in the tube, the loss increasing with amplitude, the resonance curves obtained would be distorted in consequence of a decay factor increasing with amplitude; but they appear to show no distortion of this kind, in spite of the very large amplitudes used.

It would seem, therefore, that if there is any degradation, it is of a different order of magnitude altogether from that found by Mr. Hart, and it seems worth while to examine how his figures were obtained. In the first place the measured intensities are very small. At 40 cms. distance from the siren the particle velocity is only about 0.01 cm./sec., whereas the velocities at that distance with the largest amplitude

at 96 cycles/sec. given by the vibrator should be of the order 0.2 cm./sec. It seems possible, therefore, that the intensity measurements are in error.

But however this may be, the published results hardly seem to substantiate the conclusions drawn from them, according to the present writer's analysis. The calibration curve of the hot wire microphone used is not given in the Paper, but it may be derived from the theoretical "readings" given. Thus the following table gives the microphone calibration for E in arbitrary units for a "check" microammeter reading of 54:—

Distance from source, d cms.	Theoretical "reading" of microphone.	$\left(\frac{50}{d}\right)^2 = E$ (theoretical) arbitrary units.
50	28.6	1.0
60	21.2	0.695
70	15.8	0.51
80	12.3	0.39
90	10.2	0.31
100	8.4	0.25

Next, from the curves in the Royal Society Paper, the actual experimental points are read off for a "check microphone reading" of 54; i.e., at a particular intensity at the source, for different distances, and these are expressed in intensities by means of a calibration curve drawn from the above table. The values are entered below:—

d cms.	Microphone reading.	Relative intensity E	Ed^2
50	24.4	0.82	2,050
60	17.2	0.555	2,000
70	13.6	0.43	2,100
80	10.2	0.31	1,980
90	8.4	0.25	2,020
100	7.0	0.21	2,100

The actual result appears to be a striking demonstration of the validity of the ordinary assumption, as judged by the constancy of the figures for Ed^2 entered in the last column.

DISCUSSION.

Major W. S. TUCKER: The author has described an entirely new sound-producing device of great interest, but I am not convinced as to the purity of the tone obtained. Would the air adjacent to the piston receive the same displacement as the piston itself? Examination of the resonance curves quoted in the Paper suggests that the combination of source and resonator is not as efficient as might be expected. The resonance curves are not as sharp as those obtainable from a tube in which the radiation losses from the open end of the tube are the only losses to take into account. A simple calculation based on the figures of the Paper shows that the amount of sound in terms of amplitude reflected at the open end is between 90 and 95 per cent. of the incident sound. It can be calculated, however, that in a tube of these dimensions the loss in reflection must be less than 1 per cent. The resulting resonance curves would be much sharper than those depicted and it can only be assumed that there are losses within the tube which are not accounted for by radiation at the open end and that such losses must be considerable. I would suggest testing for the presence of harmonics by means of microphones suitably tuned.

Prof. E. N. DA C. ANDRADE: The author gives the amplitudes of the motion of his piston to 1/100,000 of an inch, about half the wavelength of green light. Were measurements really made to this degree of accuracy? It also seems very risky to neglect elasticity and inertia, and to assume that a static measurement of the amplitude of the piston would give the actual amplitude for a frequency of oscillation corresponding to audible sounds. The curves in Fig. 5 are not all of the same shape, as they should be if the author's contentions were correct.

Dr. E. T. PARIS said that the addition of a pipe to the vibrator seemed to introduce unnecessary complications. Why not dispense with the pipe?

Dr. E. G. RICHARDSON said that Michotte in Holland has shown that the Rayleigh disc gives results which vary with frequency, and the difference in shape of the curves in Fig. 5 might be due to such variation. The speaker had studied the attenuation of sound in a rubber tube by using a hot wire for measuring intensity, resonance being thus avoided. The source of sound was a piston working in a cylinder and actuated by a cam driven by a motor, the speed being determined stroboscopically. He could recommend the stroboscope for determining the frequency, and a phonic motor for maintaining it constant, as a result of experience.

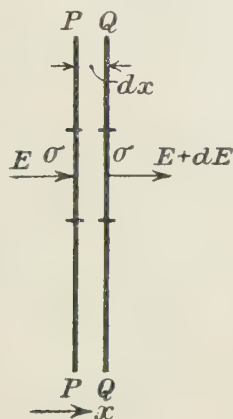
Mr. M. D. HART: I am very glad that Dr. Mallett has published the figures given in the Appendix to his Paper as it has enabled me incidentally to discover and correct an unfortunate error that appears in the figures published in my Royal Society Paper. This error, which is in the values of Ω given in the Paper in question, is explained below. It has, however, no bearing on the results deduced by Dr. Mallett in his Appendix, such as, for example, that which results in the absurdity of a rise of 103°C. in the temperature of air through which a flux of 4,500 ergs per sec. is passing. Taking a value for Ω of 3 cm.⁻¹, he points out that with this energy flux the loss per second would be 1.3×10^6 ergs per c.c., that is, that from a cubic centimetre heat energy would be produced in a given time at a rate of no less than 288 times the total energy input—an absurdity possibly even greater than that which he quotes! Such a result can, of course, only be obtained by reason of a complete misunderstanding of the meaning of the coefficient Ω . In the Royal Society Paper in question it is shown that for a spherical wave of which the energy flux at a radius r is E_r , the relation subsisting between E and r may be deduced from the definition of Ω to be

$$\frac{dE}{dr} + \frac{2E}{r} + n\Omega E = 0$$

where n is the number of periods per second of the sound. The equivalent expression for a plane wave may be obtained from this by omitting the second term and substituting for r the quantity x , a length measured in the direction in which the plane wave is progressing. We thus obtain the equation

$$\frac{dE}{dx} + n\Omega E = 0 \quad \dots \dots \dots (i)$$

a result that may also be deduced from first principles and from the definition of Ω as follows:



Let PP and QQ be two adjacent positions of the wave front separated by a distance dx . Let the energy flux at PP be E and at QQ be $E+dx$. Then the total energy passing through an area σ of the surface PP is σE and that emerging from the same area of QQ is $\sigma(E+dx)$. By definition of Ω the loss per period from the volume defined by this area and by the surfaces PP and QQ is $\Omega E \times \sigma dx$ and the loss per second n times this amount, so that

$$-\sigma dE = \sigma n \Omega E dx$$

$$\text{or,} \quad dE + n \Omega E dx = 0$$

$$\text{i.e.,} \quad \frac{dE}{dx} = -n \Omega E$$

the same equation as (1) above. It is obvious that this rate of loss only continues constant if E and Ω are invariable with x , and it is equally obvious that this cannot occur since the loss must result in the decrease of E , and hence of Ω . The amount of such decrease of E over a definite distance x cannot be deduced unless we know the

form of the relation subsisting between E and Ω . In the Royal Society Paper this relation was taken to be $\Omega = aE^p$, where p is about 1.6. Taking this value we obtain

$$\frac{dE}{dx} + anE^{1.6} = 0$$

or

$$\frac{1}{a.n.} \frac{dE}{E^{1.6}} + dx = 0$$

Integrating

$$\frac{1}{-1.6an} E^{-1.6} + x + C = 0 \quad \dots \dots \dots (ii)$$

where C is a constant. This equation, also, could have been deduced from the equivalent expression for spherical waves given in my Paper. Inserting the value of 4,500 ergs per sq. cm. per second for E when $x=0$ and putting $n=100$, the value of C becomes equal to 2.09×10^{-3} . We may then by substituting in (ii) a value for x of 1 cm. calculate what will be the corresponding value of E ; actually 100 ergs per sq. cm. per second. Thus the amount of energy available per cubic centimeter for conversion into heat is 4,400 ergs for a length of pipe of 1 cm., so that

over this length the average temperature rise would be no more than $\frac{4,400}{4,500 \times 288}$ or $\frac{1}{294}$ of that given by Dr. Mallett, resulting in a temperature rise of 0.35°C . instead of one of 103°C .

It may be emphasized that I have no reason for supposing that the value of Ω taken (i.e., 3 cm.⁻¹) is in fact appropriate to the energy flux in question of 4,500 ergs per sq. cm. per sec., and therefore that the temperature rise given above will accrue. The example has merely been worked out in order to call attention to the misconception arising in Dr. Mallett's Paper. Dr. Mallett further criticizes the interpretation of the experimental results which I obtained and has for the purpose worked out the results appearing in the two tables at the end of the Appendix to his Paper. The alleged discrepancy between his and my figures is in fact non-existent and the misconception appears to have arisen by reason of the fact that he has quoted only a particular set of experimental results and has neglected the treatment given in my Paper of the results as a whole. This treatment is given at some length and it is shown that the experimental results are satisfactorily consistent *inter se*. Without going into full details, which may be found by those interested by referring to the original Paper, it may be stated that the final result was given in the form of an equation connecting values of intensity at various different radii. From this equation, therefore, if any value of E be assumed to exist at a certain value of r , the value of E at any other value of r may be determined. If this be done for the intensity corresponding to the check microphone reading taken by Dr. Mallett it is found that the "transmission efficiency" from 50 to 100 cm. is about 98 per cent. This efficiency is defined as the ratio of the two relevant values of E/r^2 and the figures in the last column of Dr. Mallett's second table correspond to these values. It is obvious from inspection that this column of figures is obtained from experimental values which have not been smoothed out and their mean error is so great as to obscure the small reduction of 2 per cent. referred to. It is therefore not surprising that this reduction is not apparent on inspection. I do not wish to labour the point unduly but I submit that the method of dealing with the results which is described in the Paper is such that its validity is self-evident, and that the final result is the best obtainable compromise between admittedly inaccurate data obtained under extremely difficult experimental conditions. If Dr. Mallett had applied his tests by means of the final equation given instead of to particular experimental points I think it would have been clear to him that the results had not been misinterpreted.

Apart from his critique of my work, there is only one point in Dr. Mallett's Paper which raises an issue on which I should like to remark. Measurement of acoustical energy flux by means of a Rayleigh disc is analogous to measurement of electrical alternating-current power with an ammeter. The dangers which beset the experimenter who makes such measurements may best be illustrated by an example. If a stationary wave be set up the Rayleigh disc may be made to give any reading we please between zero and a maximum by placing it at given points of the wave, but at all such points the energy flux is zero. The bearing of this is that it becomes almost essential to conduct all such measurements in free air with the concomitant host of experimental difficulties. Any measurements taken in a closed room are liable to be vitiated unless the walls of the room be completely absorbent, a Utopian state of affairs which, unfortunately, cannot be realized at present.

I may now revert to the error referred to in the first sentence of my reply. When the experiments in question were first performed it was intended to define the Coefficient of

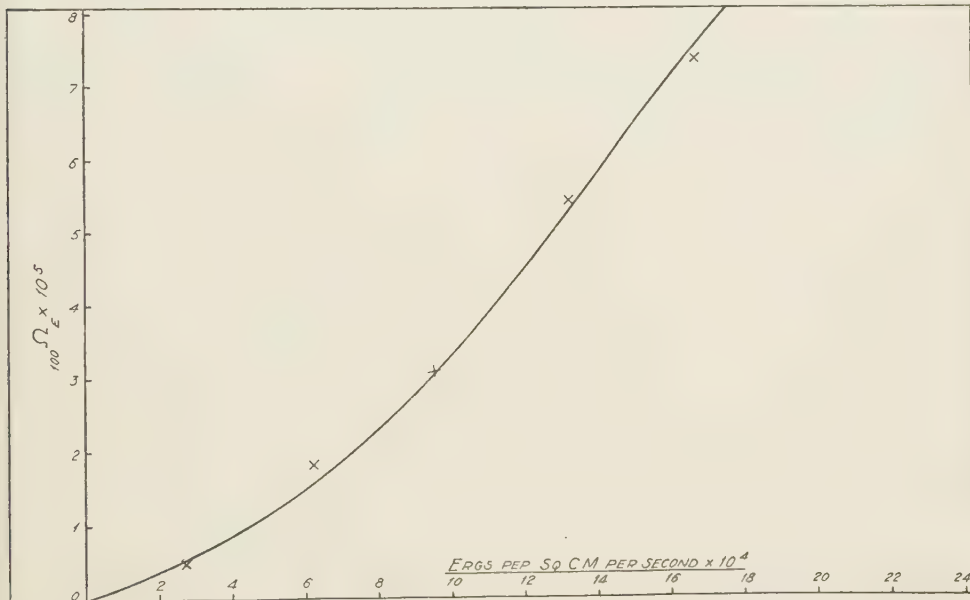
Degradation in terms of wavelength instead of period ; this decision was subsequently altered, and I regret that the necessary alteration in some of the figures has been missed. The mathematics of the Paper are correct, as are the dimensions given of Ω , but the values of the coefficient have to be divided by the velocity of sound expressed in centimeters per second and a similar correction has to be made in the value of one of the constants in the expression connecting Ω and E . I append a figure showing a graph of the correct relation, the equation to which is

$$\Omega = aE^{1.58}$$

in which a is equal to $1.67 \text{ grams}^{-1.58} \text{ cm.}^{-1} \text{ seconds}^{4.74}$ and also a table of corresponding values of Ω and E ; the graph, equation and table superseding the incorrect ones given in my Paper.

E (ergs per sq. cm. per sec. $\times 10^4$)	Ω (cm. ⁻¹ $\times 10^5$)
4	0.8
8	2.31
12	4.47
16	7.15

AUTHOR'S reply: The result of Major Tucker's calculations on the loss of energy in the tube is interesting. Since the resonance curve with a tube three times as long is substantially the same at the same frequency, it is clear that if there is this large loss in the tube it must be associated with the piston. It is clear also from the linearity of the curves of Fig. 4 that the proportionate loss does not increase with amplitude. I certainly imagine that the air



in contact with the piston moves as the piston moves ; the velocity of the latter is always far smaller than the velocity of sound. I have never suspected the presence of higher harmonics of considerable amplitudes ; but I hope to make further experiments.

In reply to Prof. Andrade, it certainly was unduly optimistic to insert the last figure of the measurements on the roller lifts. Another experiment, not recorded in the Paper, showed that the electromotive force induced in the coils of the telephone receiver mentioned in Section I was directly proportional to the speed. This, I think, indicates that the amplitudes measured statically are reproduced dynamically.

The only reply to Dr. Paris is that if the pipe were omitted, the sound amplitudes would be very much smaller. It would however, be interesting to have the piston working in a large baffle plate.

I cannot think that over the small range of Fig. 5 the frequency differences can affect the behaviour of the Rayleigh disc.

Mr. Hart's criticism of my temperature rise calculation is not sound. In order to get 4,500 ergs/sec./sq. cms. into the air, with degradation such as he postulated, enormously greater power would have to be supplied at the piston end of the pipe. The *least* value of the flux in the pipe would be 4,500, and taking even this minimum value as holding over a centimeter the temperature rise would be as I calculated it. Of course it would be greater and greater the nearer one approached the piston. Mr. Hart has now, however, divided his degradation coefficient values by 34,000, and it is hardly surprising that an error of such magnitude should have led to absurd results. But even his revised value of a in the expression $\Omega = aEp$ is enormously too large to apply to values of E of the order obtained in my experiments, where no "acoustical degradation" was observed. I still very much doubt whether Mr. Hart's theory and data rest upon a sound experimental basis.

XXI.—ON THE STATIONARY-WAVE METHOD OF MEASURING SOUND-ABSORPTION AT NORMAL INCIDENCE.*

By E. T. PARIS, *D.Sc., F.Inst.P.*

Received January 12, 1927.

ABSTRACT.

A description is given of apparatus employed for measuring coefficients of sound-absorption by the stationary-wave method. The apparatus differs from that used by earlier workers in the use of (1) a small tuned hot-wire microphone for determining relative pressure-amplitudes in the sound-waves; (2) the employment of a steady valve-driven source of sound with arrangements for maintaining the strength at a constant value; (3) the screening of source and experimental pipe from disturbances due to the movements of the observer. By the employment of a certain procedure the relation between the response of the microphone and the amplitude of the pressure-variation in the sound-wave is eliminated. Some examples of the employment of the apparatus for determining the coefficients of absorption at normal incidence of acoustic plasters and hair-felt are given.

§ 1. INTRODUCTION.

THE work described in this Paper arose in connexion with investigations initiated by the Building Research Board of the Department of Scientific and Industrial Research into the acoustic properties of certain materials. These investigations were concerned with testing the sound-absorbing properties of various plasters and other materials which could be used for lining the walls of rooms and auditoria for the purpose of reducing the reverberation in cases where it was in excess of that required for good acoustical conditions. The tests conducted by the Building Research Board were made by Sabine's reverberation method, and the results were expressed as percentage reductions in the period of reverberation produced by the introduction of 100 square feet of the material under test into a room of a certain size and shape.†

For some purposes, however, it was desirable that a method should be available for which considerably less material would be required than was needed for Sabine's method. It was, for example, desired to test a series of acoustic plasters in order to trace the effect of varying the proportions of the materials used in their making, and the necessity for preparing the comparatively large quantities needed for a reverberation experiment was an obvious handicap.

For this reason experiments were begun in 1923 with the object of establishing some method whereby a test of sound-absorption could be carried out on a specimen having an area of not more than one or two square feet. The method first suggested

* The work described in this Paper was carried out in the Acoustical Research Section of the Air Defence Experimental Establishment, Woolwich, with the assistance of the contribution made to the funds of that Section by the Department of Scientific and Industrial Research on the recommendation of their Physics Research Board.

† These experiments are described in "An Interim Note on Acoustic Experiments carried out for the Government of India" by Messrs. Barnett and Glanville, Building Research Board Paper No. 181, January, 1924.

was one in which resonance boxes were made up and lined with the materials to be tested, the sound-absorbing properties being deduced from the sharpness of resonance exhibited by the boxes. Several experimental and theoretical difficulties arose, however, which could not be readily overcome, and it was decided to seek for an alternative method of procuring the desired information.

This alternative was found in what is here called the "Stationary-wave Method"—a method which had previously been used for the study of sound-absorption by J. Tuma (1902), F. Weisbach (1910) and Hawley Taylor (1913). Some notes on the work of these investigators are given in § 3. The theory of the method is very simple and is briefly as follows. The specimen to be tested is supplied in the form of a flat disc, and (with suitable backing) is used to close one end of a cylindrical pipe (Fig. 1). The other end of the pipe remains open, and opposite to it is placed a source of sound of constant pitch and loudness. Sound-waves from the source travel down the pipe and are reflected back from the specimen. If the latter were a perfect reflector,

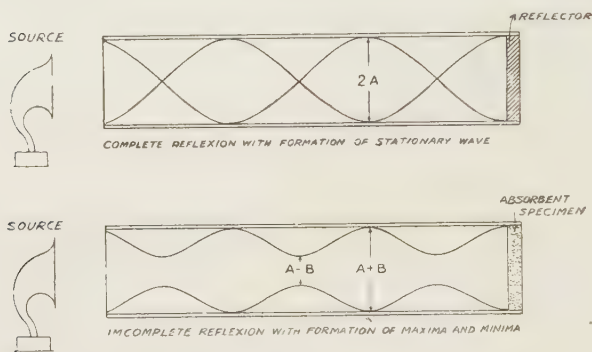


FIG. 1.—DIAGRAMS ILLUSTRATING THE STATIONARY-WAVE METHOD OF MEASURING SOUND-ABSORPTION.

the amplitudes of the incident and reflected waves would be equal and a stationary wave would be formed inside the pipe, with no pressure-variation at the loops and maximum pressure-variation at the nodes. If, however, the specimen absorbs part of the incident sound-energy, the amplitude of the reflected wave will be less than that of the

incident wave, so that instead of a stationary wave being formed inside the pipe there will now be a series of maxima and minima. If the ratio of the pressure-amplitude at a maximum to that at a minimum be determined, say a/b , the coefficient of absorption can be calculated by means of the formula (see § 2)

$$\alpha = \frac{A}{2 + a/b + b/a} \quad \dots \quad (1.1)$$

It may be noted here that the "absorbed" sound is taken to be that part of the incident sound which is not reflected. Thus, if E_i is the energy-flux in the incident waves and E_r that in the reflected waves, then the rate at which energy is being transmitted across the reflecting surface (the waves being incident normally) is $E_i - E_r$ ergs/cm.² per sec., and the coefficient of absorption is

$$\alpha = \frac{E_i - E_r}{E_i} \quad \dots \quad (1.2)$$

In the case of a porous reflecting surface, therefore, the absorbed sound includes both that which is dissipated into heat by viscous forces acting in the pores and that which is intromitted as wave-motion into the material itself.

The results so far obtained by this method indicate that it is a satisfactory one for testing the sound-absorptive properties of materials at frequencies below 700 vibrations per second, when only small specimens are available. The present Paper contains a description of the apparatus which was employed by the author and some results obtained by its use. The apparatus differs from that used by earlier workers mainly in the following particulars:—

(1) The employment of a small tuned hot-wire microphone for determining the ratio of maximum to minimum pressure-amplitude inside the pipe. This instrument is very simply constructed and easy to use, and requires no accessory apparatus beyond a battery, galvanometer and a few resistance boxes. As it is a tuned instrument, measurements of absorption at a definite wavelength can be made without the necessity of having a pure-tone source of sound.

(2) The employment of a valve-oscillator and loud-speaker as a source of sound, together with a special arrangement described in § 4(c) for maintaining the source at a constant strength.

(3) The use of a sound-chamber to enclose the source of sound and the open end of the pipe as described in § 4 (a). This is important, as the output of the source may otherwise be influenced by the movement of the observer.

Emphasis is also laid on the use of the procedure (due to Tuma) described in § 5, by which the relation between the pressure-amplitude of the sound and the response of the microphone is eliminated.

The apparatus described was designed primarily for employment with sound of a frequency of 512 vibrations per second, but a few measurements have also been made at 380 and 650 vibrations per second.

The method measures the coefficient of absorption at normal incidence under the condition that transmission is prevented as far as possible by backing the specimen with a suitable material.

§ 2. THEORY OF THE METHOD.

It is assumed that the incident and reflected sounds are plane waves travelling in opposite directions inside the pipe. In practice this condition can be secured by choosing a pipe of suitable diameter and length and arranging the source at an appropriate distance from the mouth of the pipe. Moreover, a preliminary test can always be made to ensure that a stationary wave of the required type exists inside the pipe when a good reflecting substance is used to close the distant end.

Let the axis of the pipe be parallel to the axis of x , and let the surface of the specimen to be tested lie in the plane $x=0$. The potential of the incident wave (supposed travelling in the direction x positive) can be represented by

$$\varphi = A \cos k(at - x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

After reflexion there will in general be a wave of reduced amplitude travelling in the negative direction of x . This can be represented by

$$\varphi' = B \cos k(at + x + \epsilon) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

In practice, of course, repeated reflexions take place both at the open or "source" end of the pipe and at the surface of the specimen; but this does not necessitate any modification of the above expressions, since the net effect, when a steady state is reached, is that of a single train of incident and a single train of reflected waves.

The total potential within the pipe is

$$\Phi = \varphi + \varphi' \quad (2.3)$$

If the reflecting surface at $x=0$ were perfectly rigid, the condition to be satisfied by Φ at $x=0$ would be

$$\left(\frac{d\Phi}{dx}\right)_{x=0} = 0 \quad (2.4)$$

expressing the condition that there is no motion in the plane $x=0$, so that no vibrational energy can be transmitted into the reflecting substance. Under these circumstances, the amplitudes of the incident and reflected waves are equal ($A=B$) and $\varepsilon=0$. The result is a stationary wave

$$\Phi = 2A \cos kx \cos kat \quad (2.5)$$

within the pipe.

If part of the energy of the incident sound is lost at reflexion we have

$$\Phi = A \cos k(at-x) + B \cos k(at+x+\varepsilon) \quad (2.6)$$

which, by a simple transformation ($t'=t+\frac{1}{2}\frac{\varepsilon}{a}$, $x'=x+\frac{1}{2}\varepsilon$), may be written

$$\begin{aligned} \Phi &= A \cos k(at'-x') + B \cos k(at'+x') \\ &= (A+B) \cos kx' \cos kat' \\ &\quad + (A-B) \sin kx' \sin kat' \quad (2.7) \end{aligned}$$

so that the motion within the pipe can be regarded as being due to two superimposed stationary waves, one of amplitude $(A+B)$ and the other of amplitude $(A-B)$, the nodes and loops of the first being one quarter of a wavelength distant from the nodes and loops of the second. The result is that there is within the pipe a series of positions of maximum and minimum pressure-variation proportional respectively to $(A+B)$ and $(A-B)$, spaced at distances of one quarter of a wavelength. It is the ratio of the amplitudes at these maxima and minima which is found in the experiment.

To calculate the coefficient of absorption we note that the energy-flux in the incident wave is proportional to A^2 , and the energy-flux in the reflected wave is proportional to B^2 . The fraction of the incident sound-energy lost at reflexion is thus proportional to $(A^2-B^2)/A^2$, and this is, by definition, the coefficient of absorption (α). Let the observed ratio of the maximum amplitude to the minimum amplitude be a/b . Then to find α in terms of ratio a/b we have, since

$$\frac{a}{b} = \frac{A+B}{A-B}, \quad (2.8)$$

$$\alpha = \frac{A^2-B^2}{A^2} = \frac{4ab}{(a+b)^2} \quad (2.81)$$

Or,

$$\alpha = \frac{4}{2+a/b+b/a} \quad (2.82)$$

This last expression is that given by Hawley Taylor, and is the most convenient form for computing α from the observed ratio a/b .

Alternatively, let—

$$A = (1+m)B, \quad (2.9)$$

where m is positive, since $A > B$. Then—

$$\begin{aligned} \Phi &= B \cos k(at' - x') + B \cos k(at' + x') + mB \cos k(at' + x') \\ &= 2B \cos kx' \cos kat' + mB \cos k(at' + x'). \quad (2.91) \end{aligned}$$

Alternatively, therefore, the motion within the pipe may be regarded as being that due to a stationary wave of amplitude $2B$, on which is superimposed a progressive wave of amplitude mB . The loops and nodes of the stationary wave are at the positions of maximum and minimum pressure-variation which occur when an absorbent substance is placed at the end of the pipe.

Also, according to (2.91), the positions of the maxima and minima differ from the position of the nodes and loops which would occur if a perfect reflector were used to close the pipe, by a distance $\frac{1}{2}\epsilon$. No special attempt was made to measure this shift in the experiment here described, but it may be of interest to note that it was of the order of 1 cm. for a substance with an absorption-coefficient of about 0.3 at 512 variations per second. The shift was in the direction x positive, and thus indicated a virtual node of the stationary vibration in (2.91) about 1 cm. within the surface of the absorbing material. It is possible that the measurement of this shift might be of service in investigating the mechanism by which absorption takes place in some materials. In the case of resonant reflexion it might happen that the virtual node would be in front of the reflecting surface.

§ 3. PREVIOUS EXPERIMENTS WITH THE STATIONARY-WAVE METHOD.

The first experiments with the stationary-wave method appear to have been made by J. Tuma,* in Vienna, in 1902. Later experiments were made by F. Weisbach (1910)† and Hawley Taylor (1913).‡ Taylor appears to have been unaware of the work of Tuma and Weisbach.

Tuma's observations were entirely aural. By means of listening tubes, he observed the loudness of the sound in the maximal and minimal positions in a pipe closed by absorbent materials and then found positions of the same loudness in a stationary-wave. Since the amplitude of the pressure-variation in a stationary-wave is proportional to a simple sine or cosine function, he could in this way find the ratio of the maximum to the minimum amplitude when absorption was taking place. Thus suppose the positions corresponding to minimum and maximum loudness were distant x_1 and x_2 from the nearest loop in the stationary-wave, then the ratio of maximum to minimum pressure-amplitude would be $\sin kx_2/\sin kx_1$, where $k=2\pi/\lambda$. From this ratio a quantity a was calculated such that $(1-a)$ was the ratio of the reflected to the incident amplitude. The actual results obtained were very meagre, and the interest of the Paper lies in the description of the method.

* "Eine Methode zur Vergleichung von Schallstärke und zur Bestimmung der Reflexionsfähigkeit verschiedenen Materialien," J. Tuma, Sitzungsber. d. K. Akad. d. Wissenschaften, Vol. III, Pt. 2A, pp. 402-410 (1902).

† "Versuche über Schalldurchlässigkeit, Schallreflexion und Schallabsorption," F. Weisbach, Ann. d. Phys., Vol. 33, pp. 763-798 (1910).

‡ "A Direct Method of Finding the Value of Materials as Sound Absorbers," Hawley O. Taylor, Phys. Rev., Vol. 2, pp. 270-287 (1913).

The principle of comparing amplitudes by reference to a stationary-wave has been retained in the present work, the method here described being essentially that of Tuma, with the ear and listening tube replaced by a hot-wire microphone.

Weisbach made experiments with a wooden "resonance-pipe," 238 cm. long, 25 cm. diameter, and walls 4 cm. thick. The source of sound was an electrically driven tuning-fork, mounted on a resonance-box, and in order to ensure that the source was steady, observation of the amplitude of the prongs was made by means of a microscope. The amplitude of the sound in the pipe was measured by means of a magnetophone (mounted on a slider), this being connected with a string galvanometer, the deflexion of which was taken to be proportional to the amplitude of the sound.

In the experiments of Taylor, observations were conducted with a wooden "flue," 115 cm. long, and of square cross-section (9 cm. \times 9 cm.), the specimen to be tested being held in place by means of a wooden cap. To explore the amplitude of vibration in the flue use was made of a Rayleigh disc instrument, to which the sound from inside the flue was lead by means of a long glass tube. As the Rayleigh disc involves the employment of a delicate suspension, this instrument could not be moved during an experiment, and it was therefore necessary, when comparing amplitudes at a maximum and a minimum, to move the whole of the flue together with the source of sound. This made the method very cumbersome, and it may be noted that the movement of the source during an experiment may introduce errors into the measurements, inasmuch as the output of the pipe may be influenced by its position in the room. The source of sound was a stopped organ-pipe, the note of which was purified by means of a "tone-screen," consisting of a number of resonators arranged at the open end of the flue, each resonator being designed to prevent the passage of one of the harmonics of the organ-pipe note. The absorption-coefficients were calculated from the deflexions shown by the Rayleigh disc, these being assumed proportional to the square of the amplitude of the sound. Taylor found that for 1 inch of hair-felt backed by wood, the coefficient of absorption was 0.51 at 500 vibrations per second.

§ 4. DESCRIPTION OF APPARATUS.

The description of the apparatus employed can be conveniently divided into three parts dealing with—

- (a) The experimental pipe and the mounting of the specimen ;
- (b) The hot-wire microphone and its connexions ; and,
- (c) The source of sound.

(a) *The Experimental Pipe and the Method of Mounting Specimens.*—The dimensions of the pipe finally adopted after some preliminary trials were 30.5 cm. diameter and 226 cm. length. This pipe was made from two glazed earthenware drainpipes cemented together, and was mounted, with its axis horizontal, inside a wooden box supported on four steel legs set in concrete. The pipe was insulated from its supports by pieces of hair-felt, and the box was lined with the same material. The object of the mounting was to prevent the communication of vibrations due to traffic, or other local causes, to the walls of the pipe and thence to the contained air and the microphone.

The general arrangement of the pipe and its housing is shown in Fig. 2. One

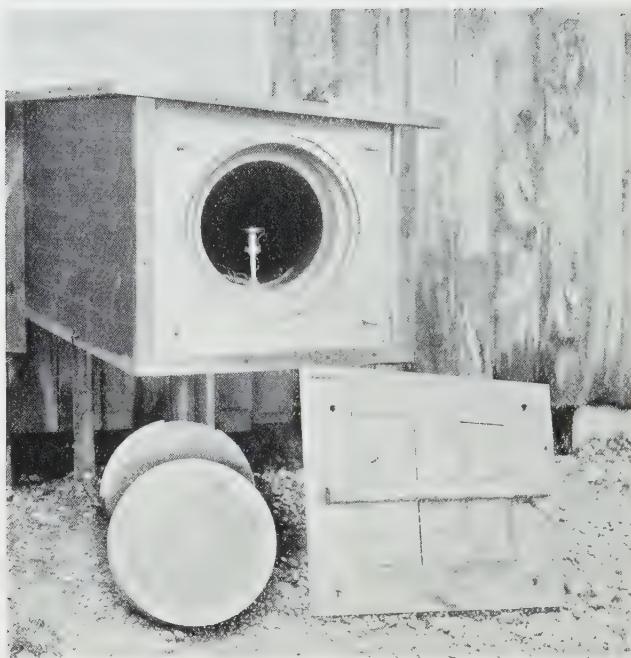


FIG. 3.—VIEW OF EXPERIMENTAL PIPE FROM
OUTSIDE LABORATORY.



FIG. 4.—SPECIMEN OF AN ACOUSTIC PLASTER
MOUNTED FOR TESTING (diameter of Plaster
Disc = 12 ins.).

[To face page 275

end of the pipe passed through the wall of the laboratory into a sound-chamber, in which was situated the source of sound, the pipe being insulated from the wall by means of felt. The object of the sound-chamber (approximately 7 ft. by 9 ft. by 7 ft.) was to prevent the output of the source from being affected by the movement of the observer in the laboratory. At the other end of the pipe, the specimen, in its mounting, was held firmly in place between the end of the pipe and the detachable cover at the end of the box (see Fig. 2). The detachable cover was held in place by four bolts and wing-nuts, and could easily be removed and replaced by hand.

The mountings for the specimens were made of discs (37 cm. in diameter), to which were screwed rings measuring 37 cm. externally, and 30.5 cm. internally. Both disc and ring were made from three layers of five-ply wood glued together. The specimens were provided as discs about 30.5 cm. in diameter, and these were fitted inside the rings, the surface of the backing provided by the disc being first covered with enough cement to fill up any inequalities in the back of the specimen.

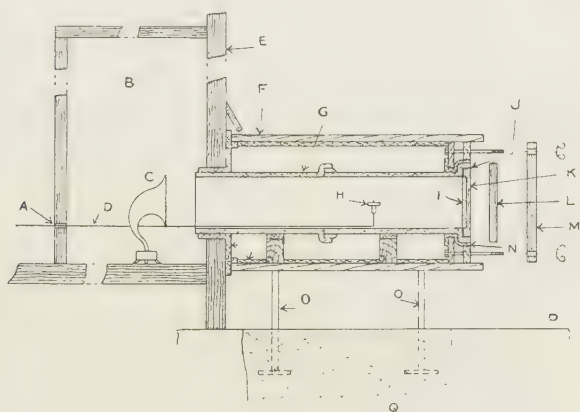


FIG. 2.—ARRANGEMENT OF EXPERIMENTAL PIPE, SOURCE AND SPECIMEN (NOT TO SCALE).

- A. Scale Read Here.
- B. Sound-Chamber
- C. Loud-Speaker.
- D. Slider.
- E. Wall of Laboratory.
- F. Felt-Lined Box Enclosing Pipe.
- G. Earthenware Pipe.
- H. Microphone.
- I. Specimen.
- J. Thin Washer of Rubber.
- K. Backing to Specimen (3 layers of 5-ply).
- L. Packing (1-inch Wood).
- M. Cover (held by Bolts and Wing Nuts.)
- N. Felt.
- O. Steel Supporting Legs set in Concrete.
- P. Ground-Level.
- Q. Concrete.

Any crevices between the specimen and the ring were filled in with plaster of Paris which was allowed to set before a test was made.

The mountings, made as described, were found to keep their shape very well, and were in every way satisfactory.

When fixing a specimen in place for a test a washer of thin leather or rubber was placed between the end of the pipe and the ring of the mounting to ensure an airtight fit.

A view of the pipe in its housing as seen from the outside of the laboratory is shown in Fig. 3, and a specimen of an acoustic plaster in its mounting ready for testing is shown in Fig. 4.

(b) *The Hot-wire Microphone.*—The microphones used in these experiments did not differ in any essential detail from the ordinary selective hot-wire microphone,* but, in order that they should offer as little obstruction as possible to the passage of the sound-waves along the experimental pipe, they were made to smaller dimensions

* *Vide* Tucker and Paris, Phil. Trans., A, Vol. 221, pp. 389-430 (1921).

than is customary for instruments tuned to the frequencies employed in the experiments. The microphone used for measurements at 512 vibrations is shown in Fig. 5. Its overall length is 6 cm., and diameter of the holder 3.4 cm. The orifice is 1 cm. long and 0.5 cm. in diameter (internal), the conductance being 0.114 cm. The container was made in two parts, which could be slid over one another, so that the microphone was tunable over a moderate range (approximately 460 to 560 vibrations per second). The damping factor at 512 vibrations per second was found to be 81 seconds⁻¹ (see Appendix I).

The microphone was carried in the experimental pipe at the end of a long wooden slider (see Fig. 2), which passed through a slot in the wall of the sound chamber. At the end which protruded through the wall of the sound chamber, the slider was provided with a millimetre-scale, so that the movement of the microphone in the experimental pipe could be recorded by an observer from outside the sound-chamber. The orifice of the microphone was on the axis of the pipe.

The electrical connexions of the microphone are shown in Fig. 6. The response was measured by a bridge method, the bridge being of the "battery" type with compensating microphone. This type of bridge is preferable to the "Wheatstone" bridge on account of its greater sensitivity, while the use of a compensating microphone makes the balance very steady. The compensating and active microphones were of the same type, but the former was, of course, shielded from the sound.

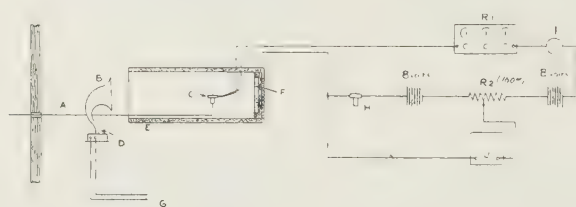


FIG. 6.—ELECTRICAL CONNEXIONS.

- A. Slider.
- B. Loud-Speaker.
- C. Active Microphone.
- D. Control Grid.
- E. Experimental Pipe.
- F. Specimen.
- G. To Oscillator.
- H. Balancing Microphone.
- I. Milliammeter.
- J. Micro-Ammeter.

The grid of the microphone carried a heating current of about 28 milliamperes, and had a "hot" resistance of about 320 ohms. When exposed to the sound the fall in the resistance was anything up to 50 ohms, depending on the position of the microphone within the tube and the output of the source of sound. This fall in resistance was measured by introducing resistance in the three-dial resistance box R_1 , reading from 0.1 ohm up to 100 ohms (Fig. 6), until the balance of the bridge was restored. The bridge was initially balanced by means of the sliding resistance R_2 .

(c) *The Source of Sound.*—The provision of a satisfactory source of sound is a matter of importance, as the source must be capable of maintaining an output of constant amplitude and pitch for considerable periods. The oscillator and loud-speaker used in the present experiments were found to be satisfactory in this respect for frequencies between 300 and 1,000 cycles per second.

During the course of an experiment it was necessary to turn the source of sound on or off several times. This was done by means of a switch, which could be used to change over the output terminals of the oscillator from the loud-speaker to an equivalent resistance.

It was found, however, that the amplitude of the sound affecting the microphone in the experimental pipe was often different before and after the source had been

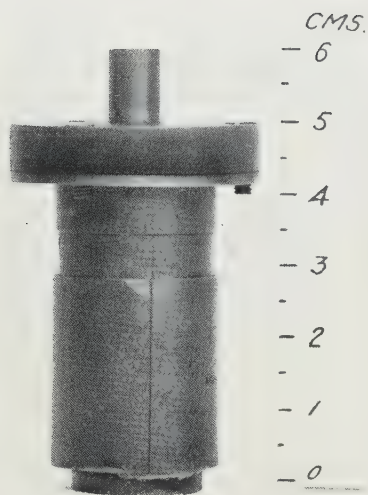
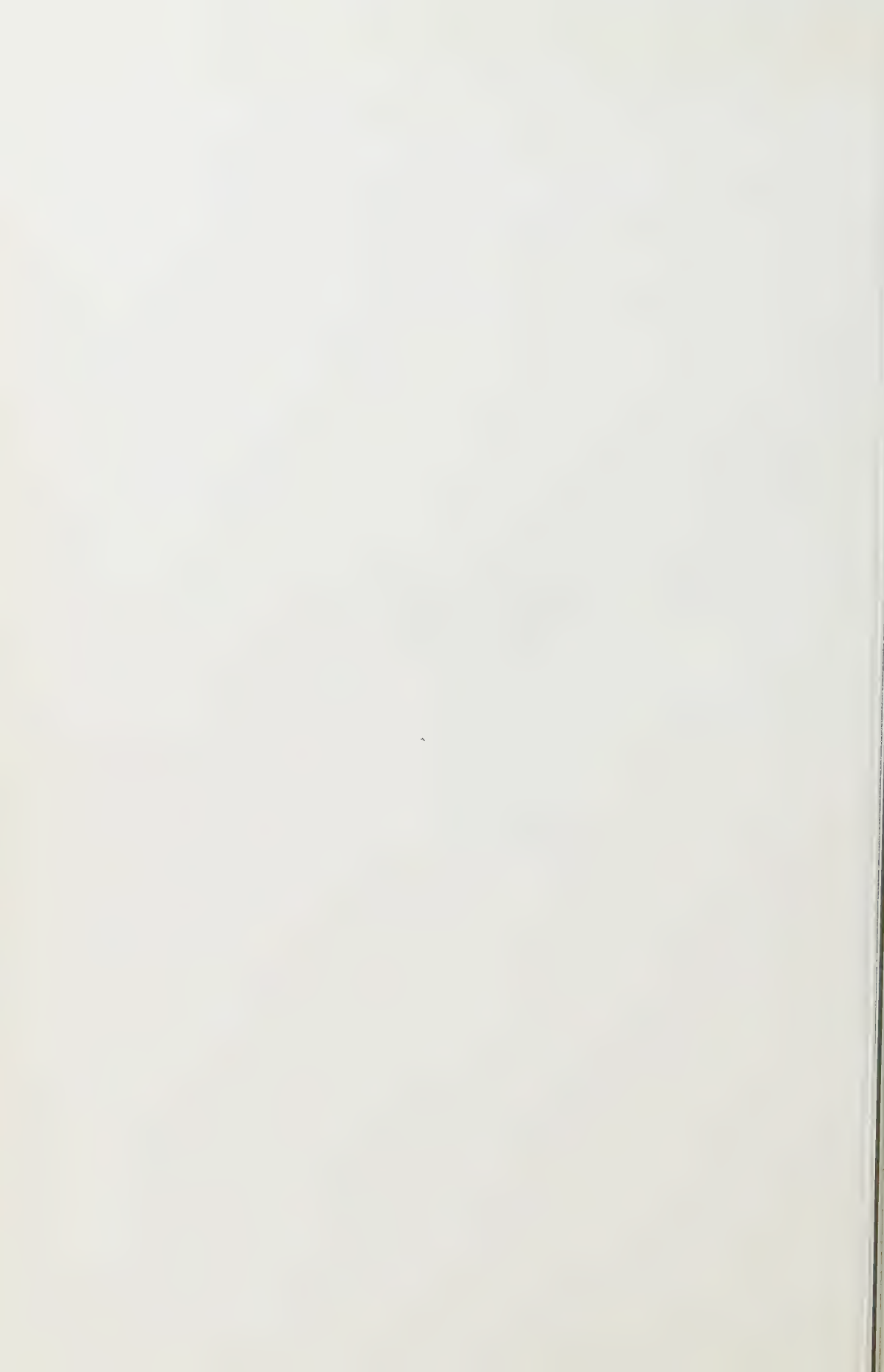


FIG. 5.—HOT-WIRE MICROPHONE
TUNED TO 512 VIBRNS. PER SEC.



switched off, and that adjusting the current in the oscillating circuit was not a satisfactory method of correcting for this variation. To overcome this difficulty and to ensure that the strength of the source remained constant throughout the period during which a set of observations was being made, the following device was made use of. A hot-wire grid of the usual pattern was mounted in the narrow end of the loud-speaker just above the diaphragm. This grid was connected to a battery-bridge of the same pattern as that used for the grid in the active microphone. At the commencement of a set of observations, the oscillator was adjusted so that this control grid showed a resistance change of about 10 ohms, and this change of resistance was maintained throughout the experiment by making small adjustments to the oscillator. This could be done without difficulty, to within about 0.05 ohm, and since experiment shows that when the resistance change in a grid is about 10 ohms, a further change of 1 ohm corresponds to a change in the amplitude of the sound of about 1.9 per cent., it follows that the amplitude of vibration of the source could be maintained constant to within about 1 part in 1,000.

§ 5. METHOD OF OBSERVATION.

The procedure adopted when taking observations was as follows. The microphone was first of all tuned as nearly as possible to the frequency of the note given by the oscillator by altering the volume of the container, a final tuning being made by a slight adjustment of the note of the oscillator. The specimen to be tested was next mounted in its place, the microphone balanced up, and the source of sound turned on. The microphone was then moved along the pipe by means of the slider shown in Figs. 2 and 6 until the position of the minimum nearest to the specimen was found. The resistance-change suffered by the microphone-grid in this position was then noted. The microphone was then moved along (away from the specimen) until a maximum was reached. The resistance-change in this position was also noted, and the specimen was then moved.

Let ρ_1 be the resistance-change at the minimum, and ρ_2 that at the maximum. It is required to find the ratio of the pressure-amplitude at a maximum to that at a minimum (i.e., a/b in the notation of § 2) from the observed resistance-changes ρ_1 and ρ_2 .

To accomplish this the experimental pipe was closed with a disc of a good reflecting substance. In the earlier experiments a piece of polished teak 1 inch thick was employed, but later this was replaced by a disc of brass $\frac{1}{8}$ in. thick, backed by three layers of five-ply wood. The position of the loop nearest to the reflector was found by sliding the microphone along the pipe. In this position, if the reflexion were perfect, there should be no change in the resistance of the microphone when the source is switched on or off. In practice the reflexion is, of course, never quite perfect, and sometimes a small change was discerned but this never exceeded 0.005 ohm at 512 vibrations per second. This residual effect at the loop appeared to be too small to be taken into consideration and it was ignored.

One method of finding a/b is to displace the microphone from the loop until it suffers a resistance change ρ_1 , the distance through which has been moved (say, y_1) being noted. It is then further displaced until the resistance change is ρ_2 , and the distance through which has been moved (say, y_2) again noted. The pressure-amplitude in a stationary-wave being proportional to $\sin ky$, where y is the distance

from a loop and $k=2\pi/\lambda$, the pressure-amplitudes producing the resistance-changes ρ_1 and ρ_2 must be proportional to $\sin ky_1$ and $\sin ky_2$, so that

$$\frac{a}{b} = \frac{\sin ky_2}{\sin ky_1}, \quad \dots \dots \dots (5.1)$$

and a can be calculated from (1.1).

In practice, however, it is easier to adjust the strength of the sound, so that $ky_2=90^\circ$, and hence, writing y instead of y_1

$$\frac{a}{b} = (\sin ky)^{-1}, \quad \dots \dots \dots (5.2)$$

and

$$\alpha = \frac{4}{2 + \operatorname{cosec} ky + \sin ky} \dots \dots \dots (5.3)$$

To ensure that $ky_2=90^\circ$, the microphone was first placed at a node in the stationary wave, and the strength of the source adjusted until the resistance change suffered by the grid was ρ_2 . The microphone was then moved into a loop position and the displacement (y) determined which was necessary to obtain a resistance-change ρ_1 . Since y is in practice often less than 1 cm., the microphone was first moved towards the specimen until the resistance change ρ_1 occurred, and then away from it until the same change was repeated, the distance between the two positions being $2y$.

The half-wavelength of the sound was found by observing the position of the second loop from the specimen.

§ 6. CORRECTION FOR THE EFFECT OF THE PRESENCE OF THE MICROPHONE IN THE EXPERIMENTAL PIPE.

It has so far been assumed that the effective pressure-variations actuating the microphone are equal to the pressure-variations which would exist in the pipe if the microphone were absent. In general, however, this will not be the case. If the pipe be narrow, the hot-wire microphone and the pipe together form a well-known type of double resonator. On the other hand, it is obvious that if the pipe be wide and the microphone very small, the coupling between pipe and microphone will be very loose, and the microphone serves merely to indicate what goes on within the pipe.

The theory of the double resonator can be utilised to calculate the effect of the presence of the microphone in the experimental pipe. Details of the calculation are given in Appendix II, where the particular case of the pipe and microphone used in these experiments for observations at 512 vibrations per second is worked out in full. The results show that for all practical purposes the effects of reaction may be ignored. The greatest effect is at a node, where the amplitude recorded by the microphone is 2 per cent. less than it would be if there were no reaction. Midway between a node and a loop the effect is about 0.9 per cent., and this decreases to zero at the loop.

It may be noted that the effects of reaction, small as they are, are largely eliminated by means of the procedure described at the end of the last section, in which, after the resistance changes ρ_1 and ρ_2 have been determined with the speci-

men in position, the microphone is placed at a node in the stationary wave and the strength of the source adjusted until the change ρ_2 again occurs.

In order to see how this procedure eliminates the effect of reaction, we may refer to equation (2.91) of § 2, which shows that, when the specimen is in position, the potential within the pipe may be regarded as being due to a stationary wave of amplitude $2B$, and a progressive wave of amplitude mB . The effect of reaction in the case of a pure stationary wave is discussed in Appendix II, and, if the small difference between x and x' in (2.5) and (2.91) be disregarded, the same theory is applicable to the stationary wave of amplitude $2B$ occurring in (2.91). The case of a progressive wave is dealt with in Appendix III, where it is shown that the effective amplitude of the potential operating on the microphone, corresponding to an undisturbed amplitude mB , due to a progressive wave is

$$mB/(1 + \frac{ac}{4h\sigma}),$$

where a is the velocity of sound in air, c is the conductance of the orifice of the microphone, h is the damping-factor of the microphone, and σ is the cross-sectional area of the pipe.

In the first part of the experiment, with the absorbing material in place, the resistance changes ρ_1 and ρ_2 are observed corresponding to *actual* amplitudes of oscillation in the neck of the microphone of, say, b_0 and a_0 . The corrected amplitude

corresponding to ρ_1 would be θb_0 , where $\theta = 1 + \frac{ac}{4h\sigma}$. Now a_0 is equal to the sum of

the amplitudes of the stationary wave and the progressive wave, say, $a_0' + b_0$. Hence the corrected value of a_0 is $\varphi a_0' + \theta b_0$, where φ is the ratio of the corrected to the uncorrected amplitude at a node—that is, 1.019 (*see* Appendix II).

In the second part of the experiment the sound is adjusted until the resistance change ρ_1 occurs at a node in the stationary wave. This therefore again corresponds to an actual amplitude of $a_0 = a_0' + b_0$, but the corrected value is now $\varphi(a_0' + b_0)$.

We wish to find the ratio of the corrected amplitudes at maximum and minimum; that is, $(\varphi a_0' + \theta b_0)/\theta b_0$, by reference to a stationary wave. By taking the reading for ρ_2 at a node we make the correction for reaction nearly the same as it was at the maximum in the first part of the experiment. For the corrected amplitude corresponding to ρ_2 is $\varphi a_0' + \varphi b_0$, which is to be compared with $\varphi a_0' + \theta b_0$, that is, we have to compare $a_0' + b_0$ with $a_0' + \frac{\theta}{\varphi} b_0$. Now $\varphi = 1.019$, approximately, and $\theta = 1.016$

(*see* Appendix III), so that, remembering that b_0 is seldom as great as $\frac{1}{5}a_0'$, and

frequently less than $\frac{1}{10}a_0'$, we see that the effect of reaction at the node is eliminated for all practical purposes.

It remains to consider the error in measuring the amplitude corresponding to ρ_1 . As shown by the figures given in Appendix II, the error involved in matching an amplitude in the stationary wave when ky is small (as it is for the amplitude corresponding to ρ_1) is quite negligible, while Appendix III shows that the effect of

reaction on the measurement of the amplitude at a minimum can be corrected for by means of the factor $\theta = 1 + \frac{ac}{4h\sigma}$. A more exact formula for the absorption-coefficient than (5.3) is therefore

$$\alpha = \frac{4}{2 + \theta \sin ky + (\theta \sin ky)^{-1}} \quad \dots \dots \dots (6.1)$$

where θ , for the apparatus used in the present experiment at 512 vibrations per second, is equal to 1.016. Here again, however, the effect is negligible for most practical purposes, as shown by the figures in the following table:—

ky	α calculated from (5.3).	α calculated from (6.1).
0°	0	0
5°	0.295	0.299
10°	504	510
15°	653	659
20°	760	766
25°	835	841
30°	888	887

The figures in the second column—that is, α as calculated from the formula (5.3)—are plotted against ky in Fig. 7. From the curve so obtained values of α can be read off directly by means of the experimentally determined values of ky .

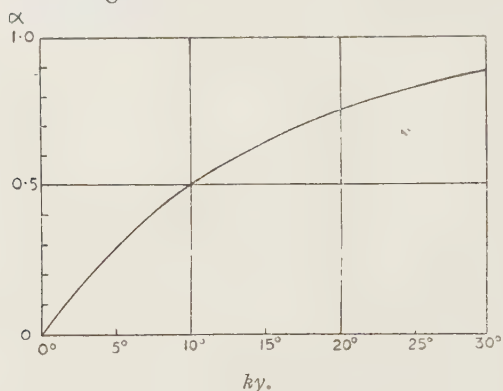


FIG. 7.—COEFFICIENT OF ABSORPTION (α) PLOTTED AGAINST ky .

§ 7. RESULTS OF OBSERVATIONS.

Some examples will now be given of measurements of sound-absorption made with the apparatus described in the preceding sections.

(a) *Acoustic Tile*.—A number of determinations have been made of the coefficient of absorption (at or near 512 vibrations per second) of samples of an acoustic tile procured by the Building Research Board from an American source. The readings obtained with this material provide an illustration of the working of the method.

The tile is a proprietary article, believed to consist mainly of pumice and Portland cement.* It has a pleasant stone-like appearance, and when used to line the interiors of halls and auditoria serves to reduce excessive reverberation. Its sound-absorbent properties are due to the porous nature of its structure. This feature is well shown in Fig. 8, which is a reproduction of a microphotograph of a section through a tile

* For this information and the microphotograph shown in Fig. 8, and also for all details of the composition and manufacture of acoustic plasters, I am indebted to the Officers of the Building Research Station of the Department of Scientific and Industrial Research at Watford, Herts.

Millimetres.

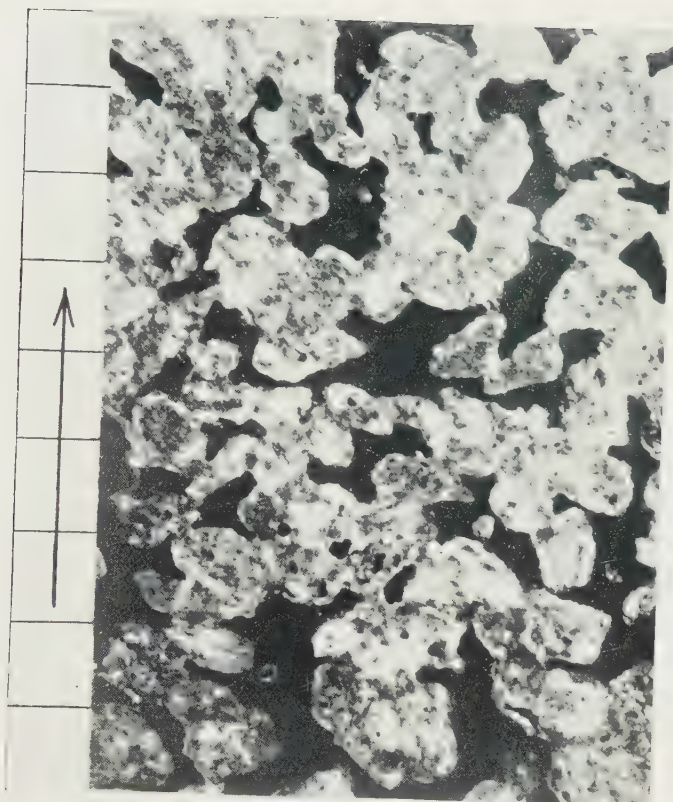


FIG. 8.—SECTION THROUGH ACOUSTIC TILE. $\times 12$.



Table of Readings for Acoustic Tile.

Experiment No.	Change of Resistance in Control Grid.		Relative Strength of Source.		Specimen in position		Reflecting plate in position. Scale-readings on slider when ohmic change in grid was ρ_1										Mean value of $2y$.	Position of 1st Loop, i.e., $\frac{3}{4}(A+B)$.	Position of 2nd Loop, i.e., $\frac{3}{4}(C+D)$.	$\frac{\lambda}{2}$	ky	α	Temp.
	ohm.	1	1	divs.	divs.	ohm.	ρ_2		Near 1st loop.				Near 2nd loop.										
							1st min.	2nd min.	A	B	$2y$	C	D	$2y$									
I				2.5 2.5	3.0	8.0			86.05 86.00 86.00	87.50 87.50 87.50	1.45 1.50 1.50	52.80 54.45 54.45	1.65 1.65 1.65	cm. 1.53	cm. 86.8	cm. 53.6	cm. 33.2	deg. 4.15	0.255	...			
II	5	1.8	14 14	14 14		28.3			86.00 86.00	87.55 87.60	1.55 1.60	52.85 54.50	1.65 1.65	1.60	86.8	53.7	33.1	4.35	0.27	...			
III	10	2.2	41 40	40 39		38.3			86.00	87.60	1.60	52.85 54.50	1.65	1.625	86.8	53.7	33.1	4.42	0.27	11°C.			
IV	15	2.7	60 60	63 62		45.0			86.00 85.95	87.50 87.50	1.50 1.55	52.90 54.40 54.40	1.50 1.50 1.50	1.51	86.75	53.65	33.1	4.11	0.25	...			

Mean value of $\alpha = 0.26$

$$n = \frac{a}{\lambda} = \frac{33,820}{66.2} = 511 \text{ vibrations per second.}$$

in a plane perpendicular to the surface which would normally receive the sound. A photograph of a section in a plane parallel to the sound-receiving surface revealed a similar structure, the average proportion of the area occupied by the pores being between 30 and 36 per cent. in each case. The tile is one inch thick.

The details of four determinations of the coefficient of absorption of a sample of this tile are given in the table on page 281.

The figures given in the second column of the table are the ohmic changes which were observed in the resistance of the control grid mounted in the throat of the loud-speaker. By removing this grid and placing it in the holder of a microphone tuned to 511 vibrations per second and calibrating it in a stationary wave, the relative strength (c.c. per sec.) of the source employed in each experiment could be discovered. The figures are given in the third column.

The figures in the seventh, eighth, tenth and eleventh columns are the scale-readings of the slider when the position of the microphone was such that the resistance-change in the grid was equal to ρ_1 . Settings were made in the vicinity of the loop nearest the specimen (readings *A* and *B*), and then, as a check, at the second loop, half a wave-length nearer the open end of the pipe (readings *C* and *D*). Generally two or three settings were made in each position, the scale being read to the nearest half millimetre, and the mean of all readings being used to find $2y$. The magnitude of the error in α likely to arise from inaccuracies in the measurement of $2y$ are discussed in Appendix IV.

A number of other determinations, some made with a different sample, gave the same mean value (0.26) for the coefficient of absorption at normal incidence of this type of tile at or near 512 vibrations per second.

This value for the absorption-coefficient is considerably smaller than that quoted for a similar kind of tile by F. R. Watson*—namely, 0.36 at 512 vibrations per second. It is to be noted, however, that the coefficient quoted by Watson was obtained by a reverberation method.

(b) *Acoustic Plasters*.—The following table contains the results of tests made on specimens of acoustic plaster submitted by the Building Research Board in 1924. Determinations were made of the absorption coefficients of each specimen at three frequencies—viz., 380, 512 and 650 vibrations per second. Each value given in the table is the mean of four or five separate determinations.

Description of specimen.	Coefficient of absorption.		
	$n=380$	$n=512$	$n=650$
Acoustic Tile 2A	0.13	0.26	0.31
Acoustic plaster 4A	0.10	0.20	...
Acoustic plaster 5A ($3\frac{1}{2}$ parts slag to 1 part magnesite)	0.18	0.25	0.27
Acoustic plaster 6A ($4\frac{1}{2}$ parts slag to 1 part magnesite)	0.18	0.27	0.30
Acoustic plaster 7A ($6\frac{1}{2}$ parts slag to 1 part magnesite)	0.21	0.31	0.36

All specimens tested were 1 inch thick, and were backed by a thin layer of cement, and either 1 inch of teak wood or three layers of five-ply.

* "Acoustics of Buildings," p. 25.

The acoustic tile 2A was the same as that just described, and its coefficients are reproduced in the table for comparison with those of the acoustic plasters.

The plaster 4A was made at the Building Research Station to a specification provided by the Sabine Laboratories in America. The constituents are granulated blast furnace slag and magnesium oxychloride cement. The slag and cement are beaten up with water to a frothy consistency, the included bubbles giving the material its porous and sound-absorbent character. The specification to which this particular specimen of plaster was made has since been considerably modified and improved by the inventor.

The plasters 5A, 6A and 7A are modified forms of the Sabine plaster, and are the subjects of a patent held by the Building Research Board. The constituents are magnesium chloride, granulated slag, magnesium oxide, lime, powdered aluminium and glue. These constituents are beaten up to a froth, the formation of which is assisted by the evolution of gaseous bubbles by the reaction between the lime and the powdered aluminium.

The samples tested show the increase in absorption-coefficient caused by increasing the proportion of slag.

(c) *Effect of Varying the Thickness of Acoustic Plasters.*—Three specimens of the acoustic plaster 6A were provided by the Building Research Board. These specimens were identical as regards composition, but had different thicknesses—namely, 1, 1½ and 2 inches. The results of tests at 512 vibrations per second were interesting as showing that thickness has a marked influence on the absorption-coefficient. The figures obtained were as follows :—

Thickness of specimen. Inches.	Absorption-coefficient.
1	0.28
1½	0.59
2	0.67

It is clear, therefore, that in making comparative tests it is of the utmost importance to see that the thicknesses of the specimens and their backings are precisely alike.

(d) *Effect of Distempering an Acoustic Plaster.*—It is clear that, since acoustic plasters owe their sound-absorbent properties to their porous character, any treatment with paint, varnish or distemper which tends to close up the pores will reduce the amount of sound absorbed. To verify this conclusion, specimens which had been treated with distemper were provided by the Building Research Board. Tests were made at two frequencies ($n=256$ and $n=512$), with the results shown below.

Specimen.	Coefficient of absorption.	
	$n=256$	$n=512$
14A Normal	0.16	0.29
15A One coat of distemper ...	0.06*	0.13
16A Two coats of distemper ...	0.05*	0.11

* These figures are to be regarded as upper limits to the coefficients, since, on account of the small amount of sound absorbed, the measurements became very difficult to take.

(e) *Absorption of Sound by Hairfelt*.—The amount of sound absorbed by various thicknesses of hairfelt has been the subject of measurement by Sabine and others, and observations made on two samples of this material are included here. It is obvious, however, that a material such as this is liable to vary considerably in structure and composition, and that no very marked agreement between the observations made by different workers is to be expected.

The felts employed in the tests were cut into circular pieces 12 in. in diameter. A mounting was used similar to that employed with the acoustic plasters, and a preliminary test was made with the mounting empty to ensure that there was no leak of sound. One disc of hairfelt was then fitted in to the mounting, and the absorption-coefficient measured. The measurement was then repeated with two, three or four layers of felt.

The two samples of hairfelt, which will be referred to as "A" and "B," differed somewhat in texture and thickness, though both were nominally $\frac{1}{2}$ in. thick. The following particulars were noted.

Sample A.—Soft, light-coloured felt. Average thickness 0.46 in. (mean of six measurements). Weight per square foot, $8\frac{1}{2}$ oz.

Sample B.—Brown felt, rather coarser texture, and harder than A. Average thickness 0.41 in. (mean of six measurements). Weight per square foot, $7\frac{3}{4}$ oz.

The sample A was tested at 512 vibrations per second, and sample B at 380 and 512 vibrations per second. The results of the tests are given in the following table :—

Hairfelt.

Number of layers.	Sample A.		Sample B.		
	Approximate thickness.	Coefficient of absorption ($n=512$).	Approximate thickness.	Coefficient of absorption.	
				$n=380$.	$n=512$.
	Ins.		Ins.		
1	0.5	0.21	0.4	0.12	0.17
2	0.9	0.52	0.8	0.28	0.38
3	1.4	0.69	1.2	0.41	0.60
4	1.8	0.69	—	—	—

It may be worth noting that the figures obtained for the felt A are very similar to those found for a $\frac{1}{2}$ -in. felt by Sabine* by the reverberation method. This is shown in Fig. 9, in which the coefficients for A are plotted against number of layers employed, and Sabine's results are plotted for comparison. One point of difference may be noted. In the case of the felt A, no appreciable increase in absorption was recorded when four layers were substituted for three layers. The curve plotted from Sabine's results, however, shows that he found a definite increase in absorption.

(f) *Cotton Waste*.—The most absorbent surface which has so far been tested consisted of a thickness of 4 in. of loosely packed cotton-waste, held in place by wires stretched across it. This material has been used by Mallett and Dutton to line a sound-chamber in which acoustical experiments were being performed, and they claimed that it was "sufficiently sound-absorbent to prevent any trouble

* "Collected Papers," p. 213, Fig. 4.

from reflected waves."* Measurements by the stationary-wave apparatus at 512 vibrations per second gave the coefficient of absorption as 0.91.

§ 8. CONCLUSION.

The stationary-wave method appears to be one which is well suited to the measurement of absorption-coefficients at normal incidence, when only small specimens of the absorbing material are available. The apparatus described in this Paper is simple and easy to handle, and measurements of absorption can be made very rapidly, the average time required for determining a coefficient, when once the specimen has been prepared, being about 20 minutes. The advantages of the particular instrument (hot-wire microphone) used for comparing the sound amplitudes are its simplicity and easy manipulation, and also the fact that, on account of the microphone being tuned, attention is confined to one particular wavelength. Another advantage is the very small area of the actual receiving surface of the microphone—that is, the cross-sectional area of the orifice. It may be noted that although the reaction of the microphone due to its resonant character has been investigated, no account has been taken of the disturbance which may be caused by the reflexion of sound from the external surface of the microphone. At the frequencies used in the experiments any effect of this kind is probably quite negligible, but at higher frequencies it could not be overlooked. It seems possible that a hot-

wire instrument could be constructed which would be sufficiently sensitive for employment up to 1,000 vibrations per second, and yet have dimensions small enough compared with the wavelength for the effect of external reflexion, or scattering, to be ignored. Above this frequency, however, the difficulty of making a small enough instrument rapidly increases, and the same trouble appears to manifest itself with other types of microphone. A possible

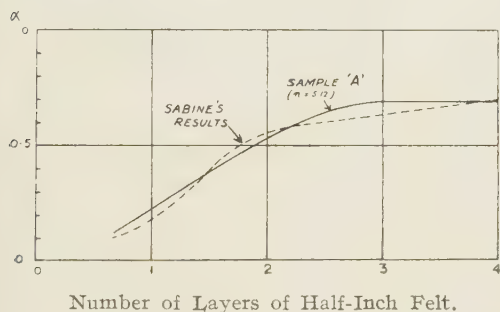


FIG. 9.—ABSORPTION OF SOUND BY HAIRFELT.

solution of the difficulty might be found in the employment of a Rayleigh disc, which could be moved about within the pipe. I have tried a disc mounted near the middle of a fine fibre attached at each end to a light metal frame, but so far have not succeeded in obtaining sufficient sensitivity for the arrangement to be of any practical use. A possible alternative is apparatus of the type used by Hawley Taylor, but the disadvantages of moving specimen and source during a set of observations are very great; also it seems likely that the employment of a long sound-duct to lead the sound from the experimental pipe to the measuring instrument may introduce complications into the theory of the method.

NOTE ADDED JANUARY 27, 1927.

The greater part of the work described on the foregoing Paper was completed in the early part of 1924, and was summarised in Report No. 256 (July, 1924), and Report No. 261 (August, 1924) of the Signals Experimental Establishment, Woolwich.

* Journ. Inst. El. Eng., Vol. 63, p. 504 (1925).

Since that time the work remained practically in abeyance (owing to the demands of other experiments) until the end of 1926, when the above account was written up, and a few additional observations made. At the time of writing I had not seen a Paper by E. A. Eckhardt and V. L. Chrisler, which appeared in April, 1926 ("The Transmission and Absorption of Sound by Some Building Materials," Scientific Papers of the Bureau of Standards, No. 526). In this Paper are given the results of measurements of sound-absorption made with a stationary-wave apparatus resembling that employed by Hawley Taylor (*loc. cit. supr.*). The source of sound was a loud-speaker, and the sound-measuring instrument a telephone, to which the sound was led by a long, narrow exploring tube. The relative amplitudes of the sounds observed were found by comparing the E.M.F.s generated in the telephone by Karcher's method (Sci. Papers of the Bureau of Standards, No. 473, 1923). An advantage of this form of sound-receiver appears to be that it can be used with very high frequencies, Eckhardt and Chrisler quoting absorption-coefficients up to 2,890 vibrations per second, and, in one case, up to 3,210 vibrations per second. The only materials for which they give absorption-coefficients which can be compared with those obtained with the apparatus described above are "Akoustolith" (i.e., the acoustic tile 2A) and 1 in. of hairfelt. For Akoustolith they gave $\alpha=0.301$ at 569 vibrations per second, which lies between the values 0.26 at 512 vibrations per second, and 0.31 at 650 vibrations per second, given in § 7 (b). It is, however, rather greater than would have been expected, and this may be accounted for by the fact that the specimen appears not to have been backed by a reflecting substance. For hairfelt (1 in. backed by a reflector) they give 0.33 at 297 vibrations per second. This again is rather higher than would be expected, but a comparison is not of much value in the case of this substance.

There is one respect in which the procedure adopted by Eckhardt and Chrisler in making observations differed notably from that described above. A different experimental pipe was used for every frequency employed in the tests, the length of the pipes being such that there was resonance to sound of the particular frequency with which the observations were made. This procedure appears to me to be quite unnecessary and to arise from an incomplete understanding of the theory of the method. The reason given by Eckhardt and Chrisler is, in effect, the following: They say that sound which enters from the source, after passing down the pipe and being reflected from the specimen, returns to the source end, where it may be again reflected with some loss of energy, and that this process may be repeated many times. This, they appear to believe, will vitiate the measurements except in two special cases: (1) When the pipe is in resonance with the sound, and (2) when there is practically no reflexion at the input end. The second case being difficult to realise experimentally, they adopted the first alternative, and constructed pipes to resound to the sound at every frequency at which measurements were made. It should be obvious, however, that when a steady state is reached, the direct and reflected waves coming from the input end will all be reflected by the specimen in the same way and that the net effect is the same as that of a single train of incident and a single train of reflected waves. The *form* of the potential within the pipe is thus determined solely by the reflecting properties of the specimen under test. The *amplitudes* of the pressures and motions, however, are dependent on the length of the pipe and the amount of energy lost at reflexion at the input end. A formal demonstration is not difficult.

Eckhardt and Chrisler also deduce formulæ for calculating the coefficient of absorption when there is loss of energy from the sound during its propagation in the pipe itself. The necessity for taking this loss of energy into consideration appears to arise from the fact that somewhat narrow pipes were used. No dimensions whatever are given in their Paper, but they state (p. 58): "In order to get a tube to resonate satisfactorily its diameter must be relatively small." In the apparatus described above, no appreciable absorption was observed in the pipe itself (*see*, for example, the observations recorded in the table in § 7).

APPENDIX I.

THE DETERMINATION OF THE DAMPING FACTOR OF A HOT-WIRE MICROPHONE.

The stationary-wave apparatus provides an easy means for determining approximately the damping factor of a hot-wire microphone. The procedure is as follows:—

The microphone is placed inside the sound-chamber with the source working steadily at the frequency at which it is desired to measure the damping factor. A series of observations is then made of the ohmic changes in the grid corresponding to different settings of the sliding part of the container. Let Q_1, Q_2 , etc., be the internal volumes of the container corresponding to the ohmic changes ρ_1, ρ_2 , etc.—the volume being determined in the usual way by weighing the container empty and full of water. Also let Q_m denote the volume for which ρ is a maximum, say, ρ_m .

The microphone is next placed in a stationary wave in the experimental pipe, and the positions found at which the ohmic changes are $\rho_1, \rho_2 \dots \rho_m$, etc. If these positions are distant $y_1, y_2 \dots y_m$, etc., from the nearest loop, then the amplitudes of the oscillatory air-currents in the neck of the microphone producing the ohmic changes $\rho_1, \rho_2, \dots \rho_m$, etc., are proportional in $\sin ky_1, \sin ky_2 \dots \sin ky_m$, etc.

From the theory of the Helmholtz resonator we know that the amplitude of the air-current in the neck, corresponding to a primary potential φ_0 , is

$$q_0 = \frac{c}{(\Delta^2 + 4h^2)^{\frac{1}{2}}} |\varphi_0|,$$

where c is the conductance of the neck, h is the damping factor, and

$$\Delta^2 = 4\pi^2 n_0^2 \left(\frac{n_0}{n} - \frac{n}{n_0} \right)^2$$

n_0 being the resonance frequency of the resonator and n the frequency of the sound. Hence, if q_a and q_b are the amplitudes corresponding to two different values of n , we have

$$\frac{q_a^2}{q_b^2} = \frac{\sin^2 ky_a}{\sin^2 ky_b} = \frac{\Delta_b^2 + 4h^2}{\Delta_a^2 + 4h^2}$$

so that

$$h = \pm \frac{1}{2} \left\{ \frac{\left(\frac{\sin ky_a}{\sin ky_b} \right)^2 \Delta_a^2 - \Delta_b^2}{1 - \left(\frac{\sin ky_a}{\sin ky_b} \right)^2} \right\}^{\frac{1}{2}}$$

If we put $\Delta_a = 0$ (the case for exact tuning) then $\rho_a = \rho_m$, and $y_a = y_m$, and we have

$$h = \pm \frac{1}{2} \Delta / \left\{ \left(\frac{\sin ky_m}{\sin ky} \right)^2 - 1 \right\}^{\frac{1}{2}}$$

where Δ and y correspond to any other tuning of the microphone, the upper or lower sign being closer according as Δ is positive or negative.

In the expression for Δ it is to be remembered that n is constant, and equal to $\frac{a}{2\pi}\sqrt{c/Q_m}$, by Rayleigh's formula, a being the velocity of sound in air; while n_0 is varied according to the setting of the sliding part of the container, and is equal to $\frac{a}{2\pi}\sqrt{c/Q}$. Thus

$$\Delta^2 = \frac{a^2 c}{4\pi^2 Q_m} \left\{ \left(\frac{Q_m}{Q} \right)^{\frac{1}{2}} - \left(\frac{Q_m}{Q} \right)^{\frac{1}{2}} \right\}^2$$

It is obvious that when Q is near to Q_m , the errors in finding Δ may be large.

The following figures were obtained by this method with the 512-frequency microphone used for the absorption-tests.

Volume of container.	ky	$\frac{1}{2}\Delta$	h
c.c.	Degrees.	sec. ⁻¹ .	sec. ⁻¹ .
11.39	6.45	337.5	79
11.28	10.5	244.5	86
11.575	12.1	214.5	73
11.93	15.05	155	83
12.325	20.15	104	84
12.60	27.7	69	109
12.96	32.8	22.5	132
13.15	33.3 ($=ky_m$)	0	—
13.25	32.1	13	51
13.565	27.3	49.5	75
13.86	20.7	84.5	71
14.49	13.95	188	76
15.33	10.5	246	84

When $\frac{1}{2}\Delta$ is small, the results are somewhat erratic, as would be expected. If these values of h for which $\frac{1}{2}\Delta$ is less than 100 be excluded, the mean of the remaining seven determinations is 81 sec.⁻¹.

APPENDIX II.

THE REACTION OF A TUNED MICROPHONE ON A STATIONARY SOUND-WAVE IN A PIPE.

The theory of the reaction of the tuned hot-wire microphone on the stationary sound-waves within the experimental pipe can be dealt with as a special case of the theory of the "Boys" type of double resonator. This theory* shows that if $q_0 e^{kat}$ represents the oscillatory air-current (c.c. per sec.) in the neck of a Helmholtz resonator of the type used in hot-wire microphones, placed at a distance l within a stopped pipe of length L , due to a primary potential $2Fe^{kat}$ at the mouth of the pipe, then

$$q_0 = \frac{c}{2h - i\Delta} \frac{(\beta - 1)F}{\beta e^{-ikL} - \mu e^{ikL}} \left\{ e^{ik(l-L)} + \mu e^{-ik(l-L)} \right\} \quad . \quad (i)$$

* Phil. Mag., Vol. II., pp. 751-769 (1926).

where c =the conductance of the orifice of the resonator ;
 h =the damping-factor of the resonator ;
 $\Delta=2\pi n_0(n_0/n-n/n_0)$;
 n_0 =the resonance-frequency of the resonator ;
 n =the frequency of the primary potential ;
 β =the (complex) coefficient of reflexion of sound-waves, coming from within the pipe, at the open end ;
 $k=2\pi/\lambda=2\pi n/a$;
 a =the velocity of sound in air.

The factor μ occurring in (i) represents the "reaction" of the Helmholtz resonator on the potential of the stationary waves within the pipe. For the theory of the Helmholtz resonator shows that the value of q_0 corresponding to a primary potential $\varphi_0 e^{ikat}$ is

$$\frac{c}{2h-i\Delta} |\varphi_0|, \dots \dots \dots (ii)$$

so that (i) shows that the effective potential in the pipe is

$$\frac{(\beta-1)F}{\beta e^{-ikL} - \mu e^{ikL}} \left\{ e^{ik(l-L)} + \mu e^{-ik(l-L)} \right\} \dots \dots \dots (iii)$$

It can be shown that, in the absence of the resonator, the potential at the same place in the pipe would be

$$\begin{aligned} & \frac{(\beta-1)F}{\beta e^{-ikL} - e^{-ikL}} \left\{ e^{ik(l-L)} + e^{-ik(l-L)} \right\} \\ &= \frac{2(\beta-1)F}{\beta e^{-ikL} - e^{-ikL}} \cos k(l-L). \dots \dots \dots (iv) \end{aligned}$$

Hence, comparing (iv) with (iii), we see that the deviation of μ from unity represents the reaction of the resonator on the potential within the pipe.

The value of μ is—

$$\begin{aligned} & \frac{1 + \frac{ac}{\sigma(2h-i\Delta)} \cos k(l-L) \cdot e^{ik(l-L)}}{1 - \frac{ac}{\sigma(2h-i\Delta)} \cos k(l-L) \cdot e^{-ik(l-L)}} \dots \dots \dots (v) \end{aligned}$$

where σ is the cross-sectional area of the pipe.

In applying these expressions to the case of the tuned microphone in the experimental pipe, we note first of all that since the Helmholtz resonator which forms the tuned part of the hot-wire microphone is in tune with the sound we have $\Delta=0$. Also, it is more convenient to measure distances from the stopped end of the pipe, and we therefore put $l-L=-x$, x being the distance from the stopped end. When these alterations have been made, the expression for q_0 and μ become

$$q_0 = \frac{c}{2h} \frac{(\beta-1)F}{\beta e^{-ikL} - \mu e^{ikL}} \left\{ e^{-ikx} + \mu e^{ikx} \right\}, \dots \dots \dots (vi)$$

$$\mu = \frac{1 + \frac{ac}{2h\sigma} \cos kx \cdot e^{-ikx}}{1 - \frac{ac}{2h\sigma} \cos kx \cdot e^{ikx}} \dots \dots \dots (vii)$$

The case of no reaction is arrived at by making $c_0 \rightarrow 0$, in which case $\mu \rightarrow 1$, and

$$q_0 \rightarrow \frac{c}{2h} \frac{2(\beta-1)F}{\beta e^{-ikL} - e^{ikL}} \cdot \cos kx \quad \dots \dots \dots \text{(viii)}$$

Now, (vi) can be written

$$q_0 = \frac{c}{2h} \frac{2(\beta-1)F}{\beta e^{-ikL} - e^{ikL}} \cdot \frac{1}{2} \left\{ e^{-ikx} + \mu e^{ikx} \right\} \frac{\beta e^{-ikL} - e^{ikL}}{\beta e^{-ikL} - \mu e^{ikL}} \quad \dots \dots \dots \text{(ix)}$$

Hence, comparing (viii) with (ix), we see that, to estimate the effects of reaction on the amplitude of the air current in the node of the resonator, we must compare the modulus of

$$\frac{1}{2} \left\{ e^{-ikx} + \mu e^{ikx} \right\} \frac{\beta e^{-ikL} - e^{ikL}}{\beta e^{-ikL} - \mu e^{ikL}} \quad \dots \dots \dots \text{(x)}$$

with $\cos kx$.

It has been assumed in the above calculation that the primary potential $2F$ is constant and independent of the position of the resonator within the pipe. In the case of the apparatus as arranged for absorption measurements, however, this is not strictly true, unless special precautions are taken. The source of sound (loud-speaker) being close to the open end of the pipe is easily influenced by changes in the disposition of the apparatus within the pipe. Thus, if the source of electrical supply to the loud-speaker remained untouched, it was found, by observation of the resistance of the control grid in the throat of the loud-speaker, that the strength of the source of sound underwent changes as the microphone was moved along the axis of the pipe. These changes were quite small (of the order of 0.01 ohm), and could easily be compensated for by adjusting the electrical supply so that the resistance of the control grid was always the same when a reading was being taken.

For a numerical calculation of the effects of reaction in the case of the apparatus employed for absorption measurements at 512 vibrations per second, we have the following values for the various quantities in the expressions given above:—

$$c = 0.114 \text{ cm.}$$

$$h = 81 \text{ sec.}^{-1}.$$

$$a = 33,760 \text{ cm./sec.}$$

$$\sigma = \frac{1}{4}\pi \times (30.5)^2 \text{ cm.}^2.$$

$$L = 226 \text{ cm.}$$

$$k = 2\pi/\lambda = 2\pi \times 412/33,760 \text{ cm.}^{-1}.$$

μ is first calculated by means of (v), for $kx = 0^\circ, 15^\circ, 30^\circ$, etc., up to 90° .

kx	μ
0°	1.0724
15°	1.0625 + $i \times 0.00052$
30°	1.0500 + $i \times 0.000723$
45°	1.0331 + $i \times 0.000549$
60°	1.0163 + $i \times 0.000232$
75°	1.0045 + $i \times 0.000036$
90°	1.0000

If the calculation is continued between $kx=90^\circ$ and $kx=180^\circ$ by steps of 15° , it is found that the numerical values of the real and imaginary parts of μ are the same for 105° , 120° , etc., as for 75° , 60° , etc., but the sign of the imaginary part is reversed. Thus, when $kx=105^\circ$, $\mu=1.0045-i\times 0.000036$. The result of this is that the effects are not quite symmetrical about a loop position ($kx=90^\circ$), but the difference is very small, as will be seen from the figures given below.

The next step is to calculate $\frac{1}{2}(e^{-ikx} + \mu e^{ikx})$ and its modulus, use being made of the value of μ given in the above table.

kx	$\frac{1}{2}(e^{-ikx} + \mu e^{ikx})$	Modulus.
0°	1.0362	1.0362
15°	$0.9961 + i \times 0.0083$	0.9961
30°	$0.8874 + i \times 0.0128$	0.8875
45°	$0.7176 + i \times 0.0119$	0.7187
60°	$0.5040 + i \times 0.0071$	0.5040
75°	$0.2594 + i \times 0.0021$	0.2594
90°	0	0

It will be seen that the effect of the imaginary part is quite negligible. If the calculation is continued between $kx=90^\circ$ and $kx=180^\circ$, the sign of the real part becomes reversed.

The remainder of the calculation of reaction effects consists (*see* (x)) in evaluating

$$\frac{\beta e^{-ikL} - e^{ikL}}{\beta e^{-ikL} - \mu e^{ikL}} \quad \dots \quad \text{(xi)}$$

or,

$$\frac{(\beta - 1) \cos kL - i(\beta + 1) \sin kL}{(\beta - \mu) \cos kL - i(\beta + \mu) \sin kL} \quad \dots \quad \text{(xii)}$$

In order to do this it is necessary to decide on a numerical value for β —that is for the coefficient of reflexion of sound-waves from the open end of the pipe next to the source.

In the case of a pipe the diameter of which is small compared with the wavelength of the sound the coefficient of reflexion from a flanged open end is*

$$\beta = -\frac{1 - ik\sigma\left(\frac{1}{c_0} - \frac{ik}{2\pi}\right)}{1 + ik\sigma\left(\frac{1}{c_0} - \frac{ik}{2\pi}\right)}, \quad \dots \quad \text{(xiii)}$$

where c_0 (the “conductance” of the open end) is approximately equal to $3.8 \times$ the radius of the pipe.

The diameter of the pipe, however, in the present experiments was of the order of half a wavelength (actually 0.46 of a wavelength at 512 vibrations per second), and the above expression for β can no longer be regarded as strictly applicable.

* cf. Phil Mag., Vol. II, p. 756 (1926).

In the absence, however, of a more satisfactory theoretical expression for β it was assumed that (xiii) remained of the same form for wider pipes and that the effect of the increased wideness could be allowed for merely by choosing a different value for c_0 .

In order to find an appropriate value for c_0 the following procedure was resorted to. The proportion of sound which escaped from the open end of the experimental pipe (away from the source) was determined experimentally by the method used for measuring absorption. The results of the observation showed that the amplitude of the wave reflected from the open end was about $\frac{1}{5}$ of that of the incident wave, that is, that the modulus of β was about $\frac{1}{5}$. It was then found by trial that the value of c_0 which must be inserted in (xiii) in order to give this value of mod β was about $10R$ (R =radius of pipe).

This value of c_0 is to be looked upon as upper limit, for it is the escape of sound from the open end of the pipe next to the source of which account has to be taken, and this must be considerably impeded by the loud-speaker trumpet, and the walls and floor of the sound-chamber. It seemed probable, from an inspection of the apparatus, that about half this value, that is, about $5R$, would be nearer the truth. On account of the uncertainty as to the proper value which should be taken for c_0 , the calculation of (xii) was made for three different values of β corresponding respectively to $c_0=3R$, $c_0=5R$, and $c_0=7R$. The results, which are given in the following tables, show that the modulus of (xii) is not greatly affected by taking different values for β within the range shown in the first table.

TABLE of β .

c_0	β
$3R$	$0.3715 + i \times 0.4652$
$5R$	$0.1875 + i \times 0.3609$
$7R$	$0.1161 + i \times 0.2804$

TABLE of Modulus of (xii).

kx	Modulus of (xii).		
	$c_0=3R.$	$c_0=5R.$	$c_0=7R.$
0°	0.952	0.947	0.944
15°	0.958	0.954	0.951
30°	0.966	0.963	0.961
45°	0.978	0.975	0.974
60°	0.989	0.988	0.987
75°	0.997	0.997	0.996
90°	1.000	1.000	1.000

The final result of the calculation is shown in the following table, where $C(kx)$ is used to denote the quantity indicated in (x). $|C(kx)|$ has been calculated for the

case when $c_0=5R$; but it can easily be seen, by reference to the preceding table, what the effect would be of making c_0 equal to $3R$ or $7R$.

kx	$ C(kx) $	$ \cos kx $
0°	0.9809	1.0000
15°	0.9501	0.9659
30°	0.8544	0.8660
45°	0.7006	0.7071
60°	0.4977	0.5000
75°	0.2585	0.2588
90°	0.0000	0.0000
105°	0.2585	0.2588
120°	0.4976	0.5000
135°	0.7004	0.7071
150°	0.8539	0.8660
165°	0.9495	0.9659
180°	0.9809	1.0000

It will be seen that the greatest deviation of $|C(kx)|$ from $|\cos kx|$ is at the nodes ($kx=0^\circ$ and 180°), where it amounts to just under 2 per cent. When $kx=45^\circ$ it is less than 1 per cent., and falls to zero when $kx=90^\circ$. The influence of this deviation on the measurement of absorption-coefficients is quite unimportant, and for all practical purposes it is eliminated by the procedure described in § 5.

In calculating $|C(kx)|$, four figures have been retained in order to show the slight asymmetry which exists about the loop position ($kx=90^\circ$), due to the reversal of the sign of the imaginary part of μ which occurs when kx passes through the value 90° .

APPENDIX III.

REACTION OF A TUNED MICROPHONE ON A PROGRESSIVE WAVE IN A PIPE.

Let the microphone lie in the plane $x=0$, and let the potential of the incident wave be $\varphi=Ae^{ik(at-x)}$. The presence of the microphone will give rise to a reflected wave of small amplitude $\varphi'=A'e^{ik(at+x)}$, while the amplitude of the wave transmitted across the plane $x=0$ will be less than $|A|$. Let the transmitted wave be $\psi=Be^{ik'at-x}$.

On the assumption that the disturbance caused by the microphone causes a departure of the waves from their plane form only for a very short distance on either side of the plane $x=0$, the conditions to be satisfied by ϕ , ϕ' and ψ when $x=0$ are

[illegible]

$$\sigma \left\{ \frac{d\psi}{dx} - \frac{d}{dx}(\varphi + \varphi') \right\} = \frac{dq}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$\frac{d^2 q}{dt^2} + 2h \frac{dq}{dt} + 4\pi^2 n_0^2 q = c \frac{d\psi}{dt} (c)$$

$\frac{dq}{dt}$ is the current, directed inwards, through the orifice of the microphone. For exact tuning, (c) becomes

[illegible]

It was ascertained, for example, that when $y=0.75$ cm. a perceptible change in resistance could be produced by moving the microphone a distance of only 0.01 cm., a movement which could not be read on scale on the slider.

Errors in the readings arise principally from: (1) The difficulty of setting the position of the microphone by operating the slider by hand; (2) the unevenness of the bottom of the experimental pipe, which tended to tilt the microphone slightly in some positions, and thus gave rise to false scale-readings on the slider; (3) slight lateral movements of the slider.

The table given above indicates that the errors in α due to inaccuracies in measuring y may be considerable when α is 0.1 or under. On the other hand, when α is above 0.5 small errors in y are not of much importance.

DISCUSSION.

Dr. A. H. DAVIES inquired what type of loud-speaker had been used? The hot-wire microphone had the disadvantage that it could not be used for the higher speech frequencies, and at the National Physical Laboratory it had been found convenient to use as detector a magnetic telephone with electric circuits whose resonance could be varied at will.

Mr. R. S. MAXWELL said that Sabine had found that the coefficient of absorption varies with the intensity of the sound employed, and had taken as standard an intensity one million times as great as that for the threshold of audibility. He had an impression that Sabine's results for felt were taken as 256 cycles, whereas those of the author and others were taken at 512. If so, the quantities plotted in Fig. 9 were not comparable, since some materials have a resonance of their own and the absorption consequently varies with frequency.

Dr. E. G. RICHARDSON urged the value of non-resonant measuring apparatus. A simple hot-wire would avoid resonance and also enable the size of the pipe to be reduced. He had used the stationary-wave method with the hot-wire anemometer for measuring absorption coefficients (Proc. Roy. Soc., A, 112, 522, 1926).

Major W. S. TUCKER said that the size of the apparatus was determined by the size of the specimens to be tested. He would like to join in congratulating the author: he had been impressed by the speed with which a variety of specimens could be tested in succession.

Prof. E. N. DA C. ANDRADE: An interesting method of measuring sound intensities is used by Messrs. Siemens & Halske, of Berlin. When sound issues from a small opening there is a partial separation of the two opposite phases of the vibration, so that at certain places there is on the whole a pressure, at others a suction effect. Just in front of the opening there is formed a narrow jet of air, whose energy depends on the energy of the sound vibration, and this jet is powerful enough to blow aside a light suspended system (of no particular sensitiveness) so as to produce deflections giving a quantitative measure of the sound intensity. The jet may also be used in conjunction with a bolometer or hot-wire microphone.

Mr. L. E. C. HUGHES said that, when working with Dr. Mallett, he had measured the attenuation of sound along the axis of a loud-speaker in a region screened with cotton waste, assuming an inverse square law for the geometrical attenuation. An energy loss of 12 per cent. on reflection had been found at a frequency of 500 cycles, the loss increasing with frequency.

Dr. A. B. WOOD asked whether the specimen discs vibrate like a diaphragm? Such behaviour would have a considerable effect on absorption: the point could be tested by varying the diameter as well as the thickness of the discs.

The AUTHOR said that a loud-speaker of the electromagnetic type was used in the experiments. In reply to Mr. Maxwell, the intensity of the sound could only be varied over a limited range, and no variation of the coefficient of absorption had been detected within this range. The quantities plotted in Fig. 9 were taken from Sabine's results for felt at 512 vibrations per second. With regard to the point raised by Dr. Wood, care had been taken throughout the experiments to test the specimen-holders before the materials were inserted. No appreciable loss of energy could be observed with the holders alone, and it was inferred that any vibration of the discs as diaphragms was quite unimportant. A device of the type mentioned by Prof. Andrade was used some years ago by Mr. F. S. Player, of the Air Defence Experimental Establishment, for exploring the resonance characteristics of pipe resonators. It consisted of a fine hole in the wall of the resonator, the issuing jet impinging on a hot-wire microphone grid.

XXII.—A BALL AND TUBE FLOWMETER SUITABLE FOR PRESSURE CIRCUITS.

By J. H. AWBERY, B.A., B.Sc., and EZER GRIFFITHS, D.Sc., F.R.S., Physics Department, The National Physical Laboratory.

Received February 11, 1927.

ABSTRACT.

The Paper describes a robust form of the Ewing Ball and Tube Flowmeter, suitable for the metering of gases or liquids under pressure, as, for example, the ammonia in a refrigerating plant. The necessary pressure-tight joints for connecting the conical tube to the circuit are described, and also a device for cutting off the flow should the tube fail. An investigation is included of the behaviour of the instrument under practical conditions of pulsating flow.

A STUDY has been made recently of the Ewing type of flowmeter, with a view to adapting it for use in metering fluids under pressure (for example, the ammonia in a refrigerating machine).

This flowmeter consists of a sphere in a conical tube whose narrower end is downward; the fluid to be metered enters at the lower end, and carries the ball up the tube to a point determined by the velocity of the fluid stream. Thus the height of the ball gives a measure of the instantaneous rate of flow, provided the tube has been calibrated for that fluid.

Experience shows that, if the tube is vertical, the ball may take up various types of motion, the height being different for the same flow, according to the type.

This effect, which would be a source of difficulty in practical applications, may be eliminated by inclining the tube, in which case the ball rolls on the wall of the tube, and is restricted to one type of motion, to which, of course, corresponds one definite calibration.

An instrument designed for use with liquids under pressure is illustrated in Fig. 1. The conical tube is of heavy walled glass tubing, and is sheathed in thick steel with two longitudinal slits, so that, should the glass give way, the fragments cannot be projected about. The junctions

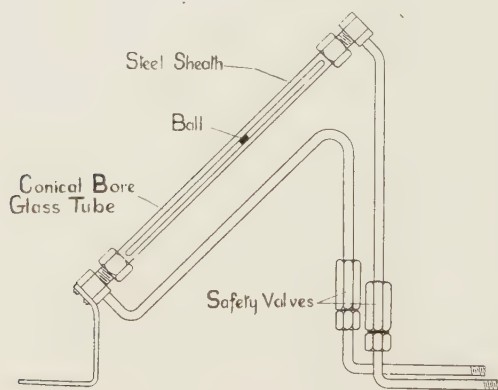


FIG. 1.

between the glass tube and the rest of the apparatus are the only other points at which special precautions are needed for dealing with high pressures.

The ends of the glass tube project slightly beyond the sheath, and are ground to a bevel edge. The steel sheath is provided with a collar, and is attached to the connecting tube by an ordinary union nut, which forces the end of the glass tube down to a lead washer, thus making a pressure-tight joint. A small metal peg projects in the centre, to prevent the ball leaving the flowmeter tube.

The device for cutting off the flow in the event of breakage is inserted in the connecting tubes, and consists of a ball valve with a relatively large chamber, the valve seating being uppermost. Normally, the ball remains at the bottom of the chamber, the liquid flowing round it, but should a sudden rush of fluid occur the ball is forced upwards into its seating, so cutting off the flow.

LIMITATIONS OF THE INSTRUMENT.

The ball and tube meter shares with the Venturi the disadvantage of having one point where the passage for the flow of liquid is greatly constricted. This involves a considerably increased velocity at this point, and therefore by Bernoulli's theorem the pressure there is much lower, so that with a highly volatile liquid, such as liquid ammonia, there would be danger of evaporation and consequently erroneous readings.

Another difficulty is one which the meter shares with all types which measure rates of flow and not total quantities—viz., if the flow varies periodically, as is the

case if the liquid is driven by certain types of pump, the position of the ball is continually varying, and the question arises as to whether the mid-point of its path may be taken for deducing the rate of flow. If there is no time lag between the establishment of a rate of flow, and the taking up of the corresponding position by the ball, it may be anticipated that the position of the ball which corresponds to the actual mean rate of flow will be the time-average of its path; this will also be true in the case of periodical variations, provided they

are truly cyclic, since the displacement-time curve for the ball will be similar to the curve giving rate of flow against time for the liquid, but displaced along the time axis (principle of forced vibrations).

An experimental study of this phenomenon has been carried out. A slow, single acting, one plunger pump was used to pump water round a circuit in which was included a ball and tube meter. The variations were much larger than would usually be encountered in practice, the period of the oscillation being 2 seconds, and the amplitude of the motion of the ball 6 inches, nearly half the length of the meter tube.

Cinematograph photographs of the ball were taken, so as to obtain a record of its position at a uniformly distributed series of times, and the film records measured up. From this a time-position curve was constructed, and is shown in Fig. 3, which is the mean of a number of successive "waves." It will be seen that the

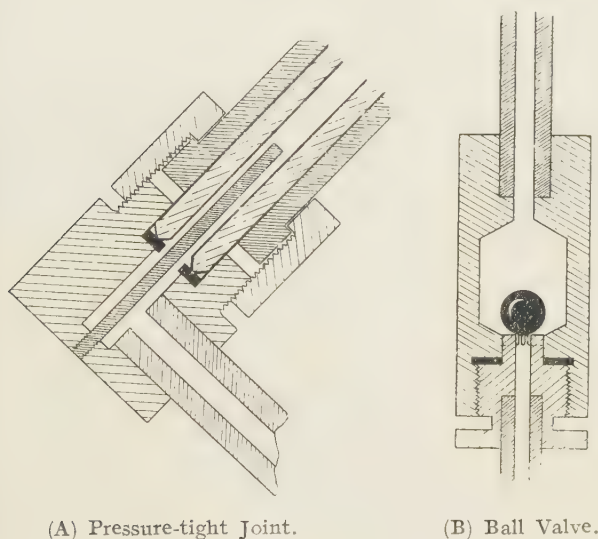


FIG. 2.—INDUSTRIAL FORM OF INSTRUMENT.

motion obtained was distinctly unsymmetrical, so that the case chosen is an extremely severe test of the error which may be introduced by this cause. Nevertheless, the following table shows that the time-average position corresponds very closely to the mean rate of flow measured experimentally, whilst the mid-position would lead to a considerable error if accepted as the reading. In general, the difference between the time-average and the mid-position would be but a small fraction of its value in this case, and no error would arise in most practical cases from a varying rate of flow.

Position of ball.	Height in tube (cm.).	Corresponding rate of flow, deduced from calibration for steady flows (c.c. per sec.).
Highest	37.2	9.3
Lowest... ..	19.15	4.3
Mid-point	28.2	6.6
True mean	26.5	6.1
(Actual rate of flow)...	—	6.0

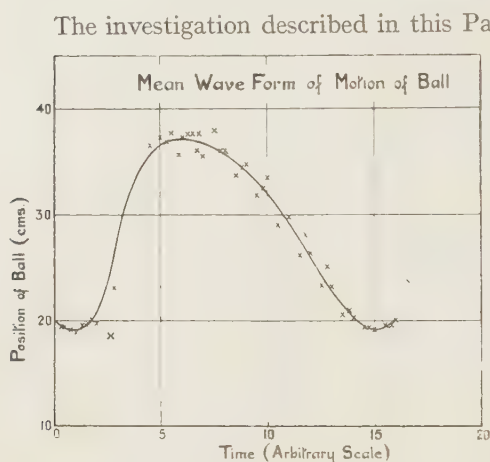


FIG. 3.

The investigation described in this Paper arose out of an inquiry by the Engineering Committee of the Food Investigation Board into the general question of meters suitable for use with refrigerating plants. The chairman, Sir Alfred Ewing, referred to this particular type of meter as one which merited further study, and we are indebted to him for his keen interest in the work. Our thanks are also due to Mr. W. F. Higgins, M.Sc., who took the cinematograph film of the motion of the ball under pulsating flow. Mr. A. Snow, observer in the Physics Department, has rendered valuable assistance with the design and

construction of the industrial form described.

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Further Experiments with the Ewing Flowmeter, by J. H. Awbery and Ezer Griffiths, Proc. Roy. Soc., Edin., December (1926).

DISCUSSION.

Prof. E. N. DA C. ANDRADE said that he had seen an instrument with a conical float, giving readings free from ambiguity, used at Heidelberg in 1911 under the name of "rotameter." The float had only one position of stability for a given velocity of flow.

Prof. F. L. HOPWOOD said that it is entertaining to watch the behaviour of a number of balls in a cylindrical tube up which water was flowing vertically. Thus two balls will remain close together and roll on one another; if three are used, the outer two remain at some distance apart

and the middle one oscillates between them ; while larger numbers arrange themselves in various other ways. If a cylinder be used instead of a ball it will set itself obliquely across the tube and revolve about the axis of the latter.

Mr. W. A. BENTON said that he had recently been interested in engineering measurements in which it had been found that the readings of a rotameter are greatly affected by changes in viscosity. The present paper would enable such instruments to be used with the security conferred by fuller knowledge.

Dr. E. H. RAYNER referred to a flowmeter at least 40 years old, consisting of a conical tube having a diaphragm perforated with a central hole, and a piston whose stem projects downwards through this hole.

Dr. GRIFFITHS' reply to the discussion : We have not had any experience with the instrument referred to by Prof. Andrade.

A useful feature of the ball and tube meter is the ease with which suitable balls can be obtained ; ball bearings are available in diameters from $\frac{1}{32}$ in. in diameter up to $\frac{1}{2}$ in. in steps of $\frac{1}{64}$ in., from $\frac{1}{2}$ in. to 1 in. in steps of $\frac{1}{32}$ in., from 1 in. to 2 in. in steps of $\frac{1}{16}$ in., etc. Balls are also available in metric sizes which help to fill in gaps between these sizes.

Then again it is a simple matter to make a hollow glass sphere of approximately the desired weight for metering gas flow. A feature of the instrument not mentioned in the Paper is the possibility of extending its range by the use of two spheres of different sizes ; the lower range is covered by the larger sphere and when this is displaced beyond the working length of the tube the smaller sphere comes into view.

In the other Paper to which a reference is given, a detailed account will be found of the theory which takes account of the effect of viscosity, etc.

The meter referred to by Dr. Rayner is extensively used for measuring water. It was invented for metering waste water a long time ago by G. F. Deacon, and improved by Lord Kelvin, who introduced the integrating device.

XXIII.—A GAS ANALYSIS INSTRUMENT BASED ON SOUND VELOCITY MEASUREMENT.

By EZER GRIFFITHS, *D.Sc., F.R.S.*, Physics Department, National Physical Laboratory.

ABSTRACT.

The instrument described is based on a principle which has not hitherto been utilised in the design of gas analysis instruments. A quartz crystal is maintained in vibration piezo-electrically, and stationary waves are set up in the gas between the flat surface of the crystal and a movable reflector. The position of the nodes is recognised by the reaction on the quartz crystal, resulting in an increase of the current in the maintaining circuit. The distance from node to node is a measure of the composition of the gaseous mixture, assuming it is composed of two gases which do not react.

THE instrument described in this Paper is based on a principle which has not hitherto been utilised in the design of gas analysis instruments. Recent work on the piezo-electric resonator by Nicolson, Cady, Pierce and Dye has shown that a quartz crystal is an excellent frequency standard when maintained in vibration by an alternating E.M.F. of the right frequency. Pierce used such a piezo-electric crystal to measure the velocity of sound in different gases by the stationary wave method.

Since the velocity of sound in air is about 332 metres per second, and that in carbon dioxide about 259 metres per second, it is obvious that, if the velocity of sound in a mixture of carbon dioxide and air can be measured with sufficient accuracy, it should afford a method of analysing the gaseous mixture. Such a physical measurement has one great advantage in that it does not involve the absorption of one constituent out of the mixture, and thus alter the composition.

It is not claimed that it is to be preferred to the simple direct chemical method for ordinary routine tests, but is simply put forward as a supplementary method for use in special cases. It might, for example, be useful in making measurements of the composition of the atmosphere in a small enclosure without opening the enclosure, when carbon dioxide and air alone are present.

The instrument to be described has the merit that its accurate functioning does not depend on the constancy of the calibration of electrical instruments, for the operation resolves itself into a length measurement—the displacement of a reflector from one position to another, which is determined by the deflection of an electrical instrument used solely as an indicator of successive maxima.

A photograph of the instrument is shown in Fig. 1, and the crystal is shown removed from the brass cylinder in Fig. 2. A diagram of the electrical circuit is given in Fig. 3.

The quartz crystal, by its longitudinal vibration, produces a train of waves in the air of frequency of about 40,000 per second, which is well above the range of audible frequency. In the present apparatus the block of quartz measures approximately $6.5 \times 3.6 \times 2$ cms. The block is lightly held between two brass electrodes, and it is brought into a state of vigorous longitudinal vibration when the electrodes are connected to an oscillatory valve circuit. The train of sound waves sent out

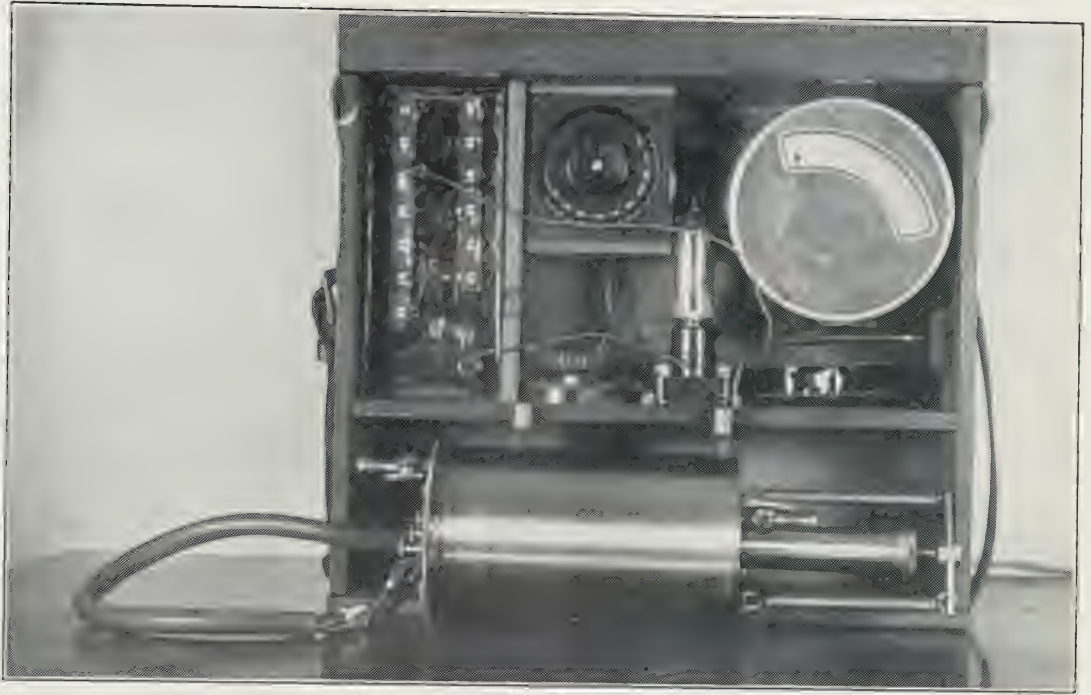


FIG. 1.

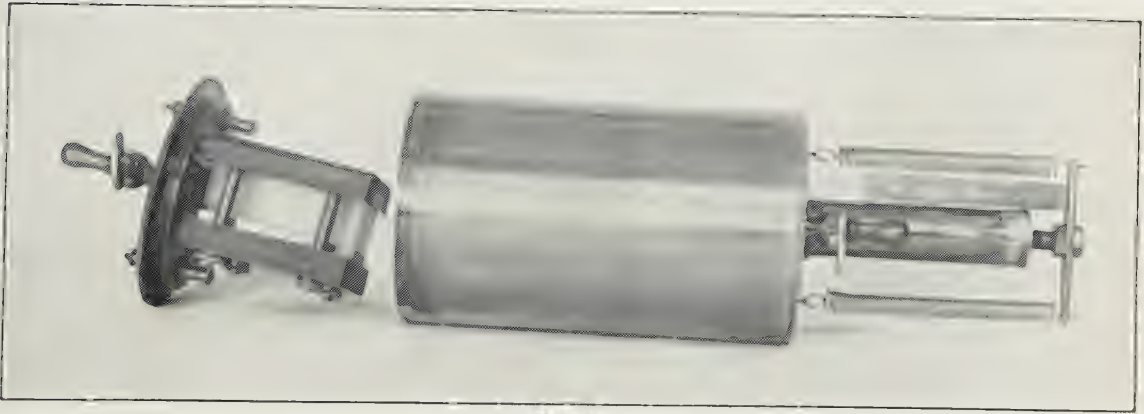


FIG. 2.

from the crystal face are reflected back from the plane face of a reflector carried, by a micrometer screw. When the reflector is at such a distance that it corresponds to a node, the reaction of the reflected wave on the crystal results in a sharply defined peak in the anode current.

To illustrate the magnitude of the change, the following results may be quoted:—When this crystal was oscillating normally there was a current of half a milliamp. flowing between the anode and the filament. This current increased suddenly to about one and a half milliamp. when the amplitude of the oscillations of the crystal

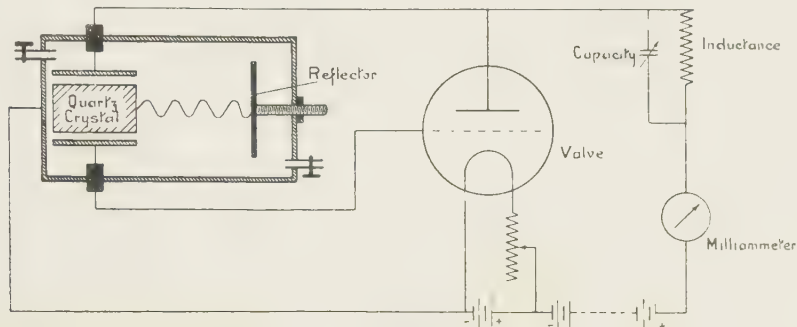


FIG. 3.

was reduced due to the reaction of the reflected wave on the crystal face. If the instrument is working normally, the position of the reflector can be defined to within one-fiftieth of a millimetre. Pierce gives a curve showing the relation between anode current and the position of the reflector, which illustrates the sharpness with which successive nodes are defined. The displacement of the reflector between

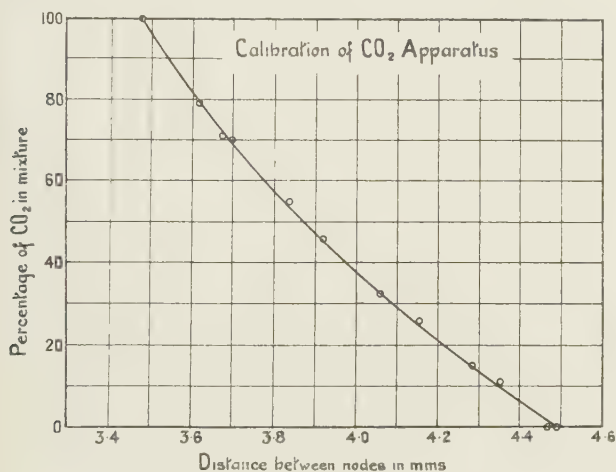


FIG. 4.

successive nodes is of the order of 4 millimetres, and the change in wavelength from carbon dioxide to air is about 1 millimetre, so an accuracy of 2 per cent. can be obtained on one setting. An inspection of Fig. 4 will show that considerably greater accuracy is obtained when the mean of several successive displacements is determined.

The present instrument has been calibrated empirically by measurements on various carbon dioxide-air mixtures, and the

calibration curve obtained is shown in Fig. 4. It will be noted that this curve applies only to the particular instrument studied, for the calibration would take account of errors in the micrometer, screw, etc.

The velocity of sound in a gas is a function of temperature, and is given by the formula

$$v = v_0 \sqrt{1 + \alpha t}$$

where v is the velocity at temperature t , v_0 is the velocity at temperature 0°C ., α is the coefficient of expansion of the gas at constant pressure.

Now for any gas $\alpha = 0.00366$ (approx.), or $1/273$ per 1°C . nearly. For air $v_0 = 33,150$ cm./sec. Hence, from the above formula

$$v = (33,150 + 61t) \text{ cm./sec.}$$

If d is the distance between successive nodes when the gas sample is at temperature t

$$d = \frac{\lambda}{2}$$

where λ is the wavelength of the sound.

Let n be the frequency, which for quartz may be taken as a constant for moderate changes of temperature.

Then $v = n = 2nd$.

So that d increases by 0.18 per cent. per 1°C . rise of temperature.

The table below shows the value of the velocity of sound in different gases (at 0°C .), and hence indicates the various combinations of two gases which can be analysed by means of a sound velocity determination, assuming, of course, that the two gases taken do not react chemically.

Gas.	Velocity, cms./sec.
Air	3.32×10^4
Hydrogen	12.86
Oxygen	3.17
Nitrous oxide	2.60
Ammonia	4.16
Carbon monoxide	3.37
Carbon dioxide	2.59*
Coal gas	4.9 to 5.15
Sulphur dioxide	2.09

One peculiarity of the instrument should be noted. The surface of the crystal has to be kept scrupulously clean and free from moisture deposit, otherwise the crystal will not oscillate satisfactorily. Consequently, the instrument is sometimes troublesome to set in operation.

The work was carried out for the Engineering Committee of the Food Investigation Board, who kindly sanctioned the publication.

In conclusion, the author desired to acknowledge his indebtedness to Dr. Dye, who kindly supplied the crystal and gave much helpful advice. His thanks are also due to Mr. A. Snow, Observer in the Physics Department, for his skill in the construction of the apparatus.

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DISCUSSION.

Dr. A. H. DAVIS asked how the author's results compare with theoretical results, such as the ratio of the specific heats of a gas. He thought that the word "sound" should not be used for supersonic disturbances. It might seem that increased accuracy could be obtained by lowering the frequency; but a limit would be set in this direction by loss of sharpness due to curvature of the wave front.

Mr. C. R. DARLING asked whether the apparatus would be sufficiently accurate for use in estimating the percentage of CO₂ in flue gases (in the neighbourhood of 16 per cent.) for the control of stoking.

Mr. J. H. AWBERY: I think I can probably answer one of the questions raised by Dr. Davis. The figures given in the Paper do not include the exact frequency of the oscillations, so that the actual values of γ , the ratio of the specific heats, cannot be calculated; but from the calibration curve it is easy to obtain numbers proportional to them. I find that if the (moist) air is taken as unity, then the 100 per cent. mixture has a γ of 0.917, and the 50 per cent. mixture one of 0.943, the question being whether these are consistent. According to the formula of Richarz, they are, since γ for the 50 per cent. mixture, calculated from the extremes, is 0.947, whereas calculated on a simple proportion it would come out to 0.964. I should like to add that this method of investigating gases seems to promise to be of use in various ways. Thus, it might be used to examine the variation of γ with pressure and temperature; arising out of this, it may even prove possible to devise a means of measuring high temperatures, since only the reflector, and not the crystal, need be exposed to the heat.

Dr. E. G. RICHARDSON remarked that a method used by Boyle in Canada for detecting the approach of ice-bergs depended on the velocity of sound in the water. In that case supersonic frequencies had to be used in order to secure a directed beam of sound.

Dr. D. OWEN expressed interest in the application of the quartz resonator. A far higher figure of accuracy, however, seemed attainable than that mentioned in the Paper. The proper procedure of measurement seemed to be to begin with the micrometer set at the zero corresponding to a resonance in air, as obtained by a preliminary set of experiments, at a distance,

say, of some 6 cm. between crystal and reflector, at some stated temperature. Then in using the instrument to determine CO_2 content the micrometer should be moved to the first resonance point, then to the second resonance point, both positions being taken as means of several settings, the temperature, of course, also being noted. The distance found between successive resonance points gives an approximate value for the half wavelength. On dividing this with the total distance between quartz and reflector a close approximation to some integer will be obtained, and on dividing the total distance by this integer a far more accurate estimate of the half wavelength results, on which the determination of CO_2 content should be based. Thus the error of setting on a resonance point is distributed over the full distance from quartz to reflector, instead of being debited to a half wavelength only, and, theoretically, a 12-fold or higher gain in accuracy is secured.

Mr. R. S. WHIPPLE suggested that the apparatus might be arranged to give a differential measurement by comparison with an air standard.

Mr. D. J. BLAICKLEY (communicated): An acoustic method of utilising the principle of the stationary wave was used in 1884 by Prof. Geo. Forbes and myself in an experimental apparatus designed for the detection of small quantities of firedamp or other gases in air. This apparatus was shown to the Accidents in Mines Commission in 1885, and exhibited in the Inventions Exhibition of that year. Briefly, the apparatus consisted of two harmonium reeds, one without a resonating tube, speaking its natural pitch; and the other, associated with a tube, speaking at a pitch varying with the density of the air passing through the tube. The difference in pitch between the simple reed and the reed-pipe (shown by beats) affords a ready and sensitive means of determining the amount of foreign gas in the air. The conclusion arrived at by the Accidents in Mines Commission, that the presence of coal-dust was an important factor in the risk of explosion, independently of fire-damp, led to the abandonment of the experiments and development of the apparatus at the time.

AUTHOR'S reply: I agree with Dr. Davis that the word "sound" is somewhat inappropriate, but the proposed substitute, "supersonic disturbance," does not appeal to me. One would naturally prefer to use lower frequencies, but this involves large quartz crystals, which are not readily procurable. It is conceivable that a steel bar or valve-driven tuning-fork could be devised to produce vibrations of sufficient amplitude and of definite frequency. In reply to Mr. Darling, the instrument could be used for the estimation of carbon dioxide in flue gases provided there is not present an unknown quantity of a third gas in which the velocity of sound is different. The apparatus referred to by Dr. Richardson is probably that patented by Prof. Langevin and M. Chilowsky; those interested in this subject should consult the article on "Echo Sounding" in *Nature*, May 9, 1925, p. 689. Special Publication No. 14 of the International Hydrographic Bureau, Monaco, August, 1926, gives a general account of the practical form of that apparatus. In reply to Dr. Owen, the procedure recommended might be employed if the apparatus could be washed out with air from time to time; but in many experiments this is inadmissible. Moreover, the fiducial setting with air cannot be determined once for all, as the crystal must not be rigidly fixed, or it will be impossible for it to vibrate longitudinally. When investigating the relation between carbon dioxide content of the atmosphere in the chamber and distance between nodes, measurements were made over about 15 half wavelengths, which virtually amounts to the procedure advocated by Dr. Owen. The utility of the instrument is determined by the difference in the velocity of sound in the two gases under consideration, and consequently it should be admirably suited to hydrogen-air mixtures for the ratio of the velocity of sound in hydrogen to the velocity of sound in air is 3.9, whereas in the case of carbon dioxide and air this ratio is only 0.77. Mr. Whipple's suggestion of modifying the apparatus so as to utilise a differential method is attractive. Possibly this could be achieved by using an electrically-driven tuning-fork, one prong being connected to the diaphragm closing one end of the gas chamber, and the other to a diaphragm of a chamber containing air. The displacement of the node with the change of composition of gas mixture might conceivably be measured by the aid of a hot wire microphone connected up by the usual bridge arrangement to a similar fixed microphone in the air chamber. I am very interested in the communication from Mr. Blaikley, as the method he describes has possibilities when a supply of the gaseous mixture can be freely drawn upon. Precautions would have to be taken with organ pipes to avoid variations in the pitch with the pressure of blowing.

XXIV.—THE SCATTERING OF X-RAYS AND THE “J” PHENOMENON.

By B. L. WORSNOP, *B.Sc.*, Lecturer in Physics, King's College,
London.

ABSTRACT.

An account is given of experiments which have been carried out using a “balance method” to obtain the “J” discontinuity which Barkla has found with heterogeneous X-rays scattered by elements of low atomic weight.

In a series of experiments, extending over several months, no such discontinuities have been found.

In view of the repeated occurrence of the phenomenon in the laboratories of Prof. Barkla, it is suggested that some condition which has not yet been published is required for its production. The intention is expressed of further investigating this effect by the method which is described in the Paper.

INTRODUCTION.

A CONSIDERATION of the work of Prof. Barkla and his collaborators* on the “J” phenomenon led the writer to perform some experiments which were designed to show the source of the electronic emission which those experimenters had found to be associated with the “J” discontinuity. These experiments, carried out early in 1926, yielded negative results, and it was then thought that this might be due to wrong conditions in the exciting radiation.

Experiments, which are here summarised, were therefore undertaken with a view to obtaining the conditions necessary for the production of the phenomenon. In these investigations as many of the conditions of the experiments of Barkla as possible were reproduced, as will be seen in the following account—e.g., similar heterogeneous beams were used, and the penetration was measured in terms of a mean absorption coefficient, which was obtained from a 50 per cent. absorption.

As is well known, one of Barkla's methods of showing the “J” discontinuity was to direct a primary beam of X-rays on to a scatterer of low atomic weight, and to compare the radiation scattered in a direction at right angles to the incident beam, producing an ionisation S in an ionisation chamber arranged to receive it, with the ionisation P produced by the direct beam which had passed through the scatterer. The ratio S/P was measured directly.

Absorbing sheets of aluminium of equal thickness were then placed in the path of both beams, and the ratio of the ionisation then produced (S'/P') was obtained.

Barkla and Khastgir found that if S/P was arranged to be of the order of unity for a soft radiation it maintained that value as the average penetration of the incident beam was increased. The ratio S'/P' , on the other hand, was found to remain constant for the softer radiation, but at certain critical penetrations, corresponding to well defined magnitudes for (μ/ρ) , the ratio S'/P' was suddenly decreased. Thus, when aluminium of 0.05 cm. was used, a 10 per cent. drop in S'/P' occurred at $\mu/\rho=3.8$, and again at 1.9 for the primary radiation.

* *Phil. Trans.* (1916); *Phil. Mag.*, November and May (1925).

APPARATUS.

For the experiments described below the apparatus was designed to detect small *differences* in the primary and the scattered beams: the general arrangement is shown in Fig. 1.

The source of X-rays was a tungsten target Coolidge tube, which was excited by a transformer, controlled by means of an independently regulated field ensuring a similar secondary wave-form for all potentials. The unrectified secondary was applied directly to the tube, and the effective potential was maintained constant for each observation by a slight change of the field current when necessary.

To detect any potential changes during an observation the following arrangement

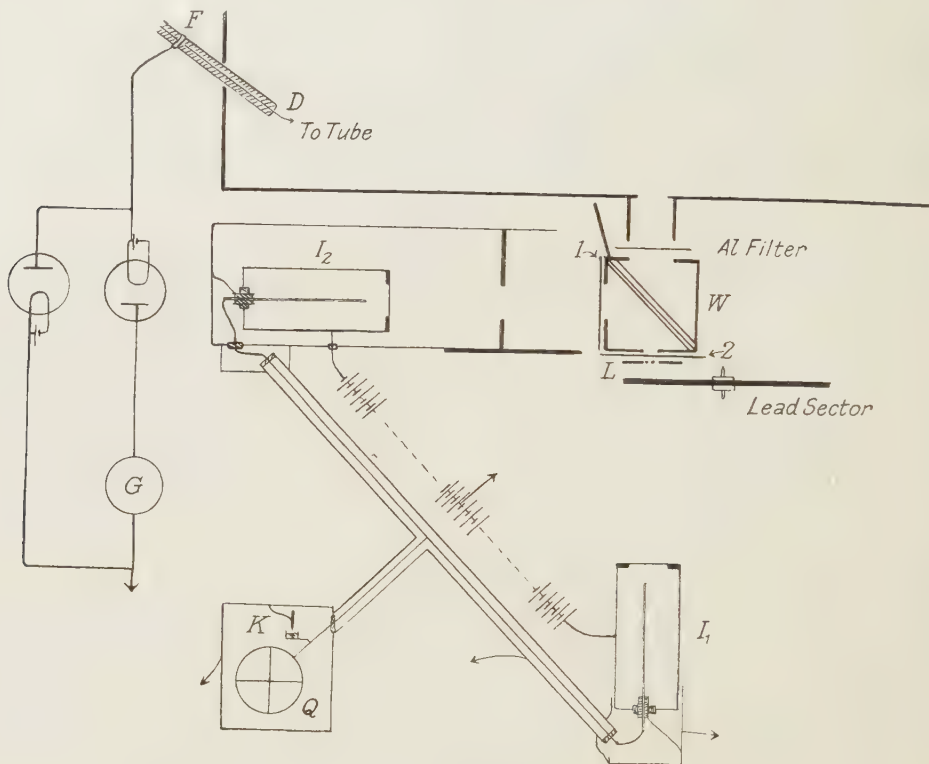


FIG. 1.

of apparatus was used. A narrow strip of tin foil, *F*, was wrapped round the thick ebonite insulator which surrounded the lead from the secondary of the transformer. This was connected to two insulated valves as shown. The induced potential due to one half of the wave sends a current through one of the valves to earth, and the potential due to the other half, which is effective in the production of X-rays, sends a current through the other valve. This current passes through the galvanometer *G*, and is very nearly proportional to the peak value of the potential of the inner wire *D*.* The galvanometer reflected a spot of light on to the same scale as that

* The writer is indebted to Mr. F. S. Robertson for information, which he has not yet published, showing that under suitable conditions this proportionality holds.

due to the quadrant electrometer, Q . Any variation in potential applied to the tube was therefore immediately detected and controlled.

The scattering substance was contained in a cubical lead box, W , of about 12 cms. side, in the first experiments. The radiation scattered at 90 degrees passed through a circular aperture, of about 5 cms. diameter, to an ionisation chamber I_2 . The primary radiation continued through a 1 cm. aperture, and a perforated lead screen, to a second ionisation chamber I_1 .

The ionisation chambers were of identical design, and were placed at a distance of not less than 20 cms. from the apertures in the lead box.

A battery of dry cells of some 360 volts was earthed at the mid-point and the two ends were connected to the outer walls of the ionisation chambers in the usual way: the collecting rods were joined together to one pair of quadrants of a quadrant electrometer, which therefore gave a deflection proportional to the difference of the ionisation in I_1 and I_2 .

In order to avoid any possible effects due to the gas in the chambers I_1 and I_2 were filled with air and closed by thin gold-beaters' skin.

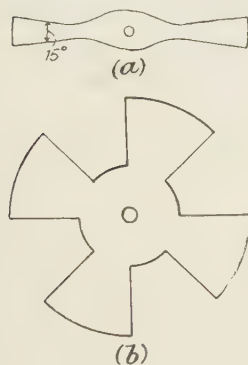


FIG. 2.

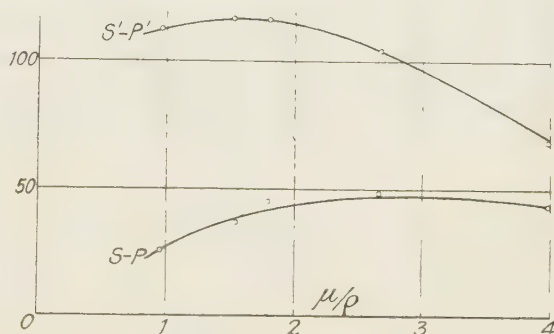


FIG. 3.

Lead sectors of the forms shown in Fig. 2 could be rotated by means of an electric motor, so that the primary beam was cut off intermittently by them.

METHOD OF WORKING.

A heterogeneous beam of X-rays from the tube was limited by lead screens and then fell on the scatterer, either directly, or else after passing through an aluminium filter.

By means of a lead screen perforated by some 10 pin holes, the primary beam was further limited so that the ionisation in I_1 was very nearly equal to that produced in I_2 by the wide beam which was scattered at right angles to the initial direction; i.e., the quadrant electrometer had a slow rate of movement corresponding to $P-S$ approximately equal to zero. Sheets of aluminium of equal thickness were then placed in the positions 1 and 2 of Fig. 1, and the process was repeated so that $P'-S'$ was obtained.

The potential applied to the tube was then changed and a similar set of observations were taken. Throughout the experiments, the filament current through the

Coolidge tube was maintained constant (at 3.7 amperes). The current through the tube was from 3.5 to 4.2 milli-amperes.

Curves showing the relation between $S - P$ or $S' - P'$ and μ/ρ were continuous, as seen in the typical example in Fig. 3. They seemed to indicate that in all the experiments undertaken, the value of S'/P' did not suddenly change.

It was, of course, realised that unless P and P' were constant (which is obviously not the case in a Coolidge tube when the potential is increased) the $S - P$ value does not give the magnitude of S/P . For this and other reasons, it appeared desirable to reduce the observations to the same form as those of Barkla, whilst at the same time retaining the advantages of the balance method. To do this and also to obtain precise values for the penetration of the rays used, the lead sectors shown in Fig. 2 were introduced.

The method of working now had the following sequence. The tube was excited by a definite potential and the quantity $P - S = x$ was measured; a lead sector of two arms, each 15° , was then rotated by a small motor, so that the rays which had passed through the holes in the lead screen, L , were cut off for $30/360 = 1/12$ of the time. It is therefore apparent that the quadrant electrometer recorded the difference between $11/12 P$ and S , equal to y , say. (x and y were measured as the number of divisions on the scale of the electrometer moved over in 30 seconds, 10 seconds after the earthing key K was raised.)

The value of P and S could therefore be obtained in terms of the rate of movement of the electrometer, and not merely as a ratio, for,

$$1/12 P = x - y; \text{ and } P - S = x;$$

and further these values are compensated for any variations during the observations, and were for the same initial beam at the same time.

The aluminium sheets 1 and 2 were then introduced and the process repeated and the value of P' and S' were similarly deduced.

To determine the mean absorption coefficient of the radiation used, the 15° sector was replaced by the one shown in Fig. 2 (b): the sheets 1 and 2 were removed and in place of 1, a sheet of aluminium of known thickness was set up in the path of the scattered rays, the motor was set in motion and so cut off half the primary beam. The electrometer therefore now measured $P/2 - S_1$, where S_1 is the residual scattered radiation which had passed through the aluminium. A second piece of aluminium sheet was then added to the first and the rate of movement again measured. This process was repeated with a sufficient number of aluminium sheets to enable a curve to be plotted showing the relation between t the thickness of aluminium and the rate of movement. The range of thickness was so chosen that the rate $x/2$ was included.

From this curve the thickness of aluminium which would cut down the rate of movement to $x/2$ was read off.

From these observations it is a simple matter to find the exact value of the equivalent μ/ρ which corresponds to a 50 per cent. absorption, for we have

$$P - S = x$$

$$P/2 - S_1 = x/2$$

where S_1 is the ionisation produced by the scattered rays which have passed through a thickness t of aluminium, such that the rate is $x/2$.

From the preceding equations

$$2 S_1 = S$$

Also,

$$S_1 = S e^{-\mu t}$$

Therefore

$$1 = 2e^{-\mu t}$$

or,

$$\mu = \frac{2.3 \log_{10} 2}{t}$$

whence

$$\left(\frac{\mu}{\rho}\right)_{ae} = \frac{2.3 \log 2}{2.7t} = \frac{0.256}{t}$$

The equivalent absorption coefficients, calculated by the above method, were thus compensated for any variations during the observations, as were P , P' , S and S' .

It will be noticed that the μ/ρ calculated is for the scattered radiation, but as this is not very different from the primary, it serves as a measure of the penetration of the radiation used.

RESULTS.

Table I is given to show the kind of results obtained. It will be noticed that the values of S/P and S'/P' are mostly consistent to 1 per cent., and that in very few cases are variations greater than 2 per cent. to be found.

TABLE I.

$\frac{\mu}{\rho}$	$P-S$ (x)	$\frac{11}{12}P-S$ (y)	$P'-S'$ (x')	$\frac{11}{12}P'-S'$ (y')	$x-y$	$x'-y'$	P	S	P'	S'	$\frac{S}{P}$	$\frac{S'}{P'}$
3.04	47	-21			68		816	759			0.93 ₅	
	44	-19			63		756	712			0.94	
			16.5	-19		35.5			426	409.5		0.96
			14	-29		43			516	502		0.97 ₄
			19	-17		36			432	413		0.95 ₅
1.96	-131	-240			109		1308	1439			1.10 ₂	
	-133	-239			106		1272	1405			1.10 ₅	
			-141	-223		82			984	1125		1.14 ₅
			-137	-232		95			1140	1277		1.12
2.2	-85	-197			112		1344	1429			1.06	
	-81	-202			121		1452	1533			1.05 ₅	
	-101	-220			119		1428	1529			1.07 ₂	
			-102	-200		98			1176	1278		1.08 ₈
			-110	-200		90			1080	1190		1.10
			-108	-189		81			972	1080		1.11
3.88	65	23			42		504	439			0.87 ₂	
	66	23			43		516	450			0.87 ₃	
			39	14		25			300	261		0.87 ₁
			39	15		24			288	249		0.86 ₈
2.82	35	-42			77		924	889			0.96	
	27	-55			82		984	957			0.97	
			12	-40		52			624	612		0.98 ₂
			9	-38		47			564	555		0.98 ₅
			4	-45		49			588	584		0.99 ₁

* For this observation the sheets 1 and 2 were interchanged.

Figs. 4(a), 4(b) and 4(c) are three curves showing the behaviour of S/P and S'/P' as the mean μ/ρ is changed. They are actual curves for three sets of observations, but also are typical of the results in general.

Curve 4(c) is for an initial beam falling directly on the scatterer; curves 4(a)

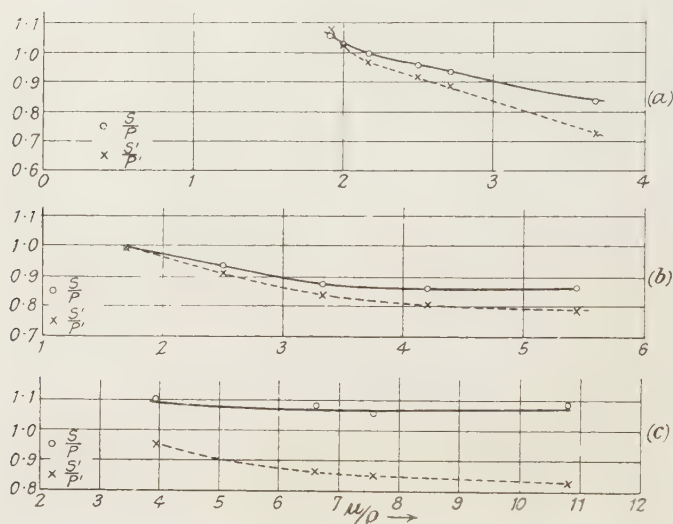


FIG. 4.

and 4(b) are for radiation which has passed through a preliminary aluminium filter before impinging on the wax.

These curves are for a scatterer placed in the "reflecting" position, as in Fig. 5 (a). It was realised that in this position the scattering from increasing depth as the incident beam became more penetrating, would cause a relative hardening of the scattered rays, but as a sudden change of the order of 10 per cent.

was looked for, this was not considered a serious disadvantage.

It will be seen that there is an upward slope in all the curves at the smaller μ/ρ end; but an examination of the curve for S'/P' in the region where the "J" discontinuities have been previously reported, viz., at $\mu/\rho = 1.9$ and 3.8 does not reveal any such change. This was the case in every experiment performed, covering in all the range of μ/ρ from 11 to 1.6.

The next set of results were similar to the above, but were obtained with the wax scatterer at right angles to the former position, as illustrated in Fig. 5(b). Here again the curves for S/P and S'/P' for different μ/ρ , slopes upwards as μ/ρ decreases, also the curves are free from discontinuities, at least to within the limits of error of observation.

Figs. 6(a) and 6(b) are typical of these results. In the former case the initial beam falls directly on the scatterer, whereas, in the latter, the rays pass through a preliminary filter of aluminium.

Finally, a set of observations was taken with the side AB (Fig. 5) of the lead box removed: this was to see if the rise in the curves at the lower μ/ρ values was due to radiation scattered from the wax to this side and then through the wax to I_2 .

The results of this set of observations are summarised in Fig. 7.

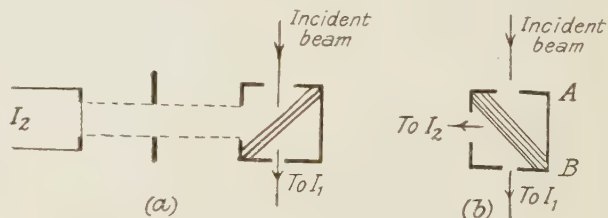


FIG. 5.

Whereas the curves do not show the same steady rise as before, they do not show any indication of the phenomenon sought: where S/P changes, the corresponding S'/P' changes in a similar way.

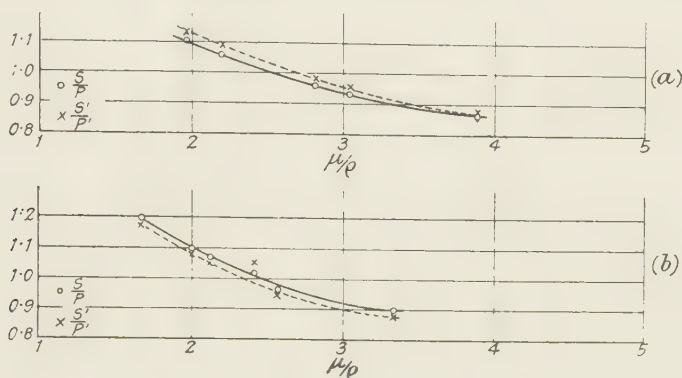


FIG. 6

SUMMARY AND CONCLUSIONS.

The above experiments are an elaboration of what was intended to be a preliminary experiment for another investigation. They were intended to obtain the correct penetration of the incident radiation to produce the "J" discontinuities which Prof. Barkla and his collaborators have repeatedly found by their method,

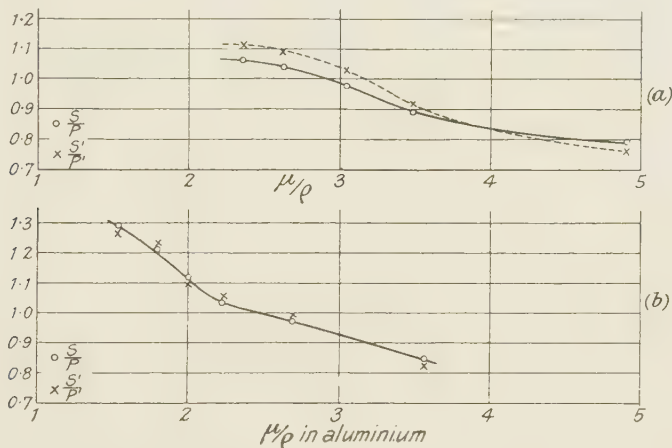


FIG. 7.

but in no experiment has a discontinuity been found, in a region which should have shown at least two jumps.*

The only conclusion to be drawn seems to be that there must be some condition

* Barkla and Watson, Phil. Mag., Nov. (1926), have found discontinuities at the following values of : 3.8, 3.24, 2.44, 1.94, all within the range of these experiments.

necessary, additional to those in these experiments, and to the published accounts. It seems somewhat premature to offer suggestions as to the nature of those conditions, but it seems desirable to apply this method to the later experiments of Barkla and Khastgir* and Barkla and Watson.† This will be done at the earliest opportunity.

It is felt that the results as expressed in the curves shown are not suitable to detect any gradually varying change such as would be anticipated from the Compton theory.

In the experiments which are to be undertaken an attempt will be made to include this, by using beams of a less heterogeneous nature.

In conclusion I would like to take this opportunity of thanking Prof. O. W. Richardson for the interest he has taken in these experiments.

DISCUSSION.

Prof. E. A. OWEN said that Mr. Dufton and he had photographed the shadow of a wedge, using both direct and scattered radiation, and had found no evidence of discontinuity on measuring the variation in intensity along the shadow. The work had been abandoned under the impression that some requisite experimental condition must have been missed; but in view of the negative result obtained by the author also, further evidence from different laboratories was desirable.

Mr. J. GUILD said that both Barkla and the author used the formula $I = I_0 \cdot e^{-\mu t}$, but, strictly, this was inapplicable to a heterogeneous beam in a selectively absorbing medium.

Dr. LEWIS SIMONS (communicated): The whole argument seems to me to turn on the somewhat doubtful assumption that the average quality of the primary rays is the same on both sides of the 10 pin-holes in the lead screen *L*. It is assumed that these holes act merely as a stop. In view of the obliquity of the primary rays in passing through these holes, it would appear that considerable transformations would occur on their walls. Have any preliminary experiments been performed to test the similarity of the primary radiation on both sides of this group of pin-holes for any given condition of the X-ray tube?

The AUTHOR, in reply to the discussion, said that he was much interested to hear that Prof. Owen had obtained negative results with a wedge. He was himself now trying the effect of equal increments of screen thickness. He used a heterogeneous beam because he wished to reproduce the conditions described by Barkla. He did not quite follow Dr. Simons' point about the pin-holes: they could scarcely account for the 10 per cent. discontinuity, and in any case they had been used by Barkla.

* Barkla and Khastgir, *Phil. Mag.*, Sept. (1926).

† Loc. cit.

XXV.—THE CHARACTERISTICS OF THERMIONIC RECTIFIERS.

By Prof. C. L. FORTESCUE.

ABSTRACT.

This Paper extends the results obtained in a previous Paper to the case of rectifying valves working at low voltages with unsaturated electron currents. The most economic conditions are briefly discussed, and it is shown that, as far as the limited information with respect to the life of the modern valves is concerned, it is probable that long life should be provided for.

INTRODUCTION.

IN a Paper published in 1919* methods were given for calculating the behaviour of thermionic rectifiers working at high voltages and employing filaments with sharply defined saturation values for the emission current. Valves are now largely used on low-voltage circuits where the saturation value of the emission current is not reached, and even with some modern designs of high power, high-voltage rectifiers, the conditions of operation are such that the total emission from the filament is considerably greater than the maximum current through the valve. Shearing† has suggested an approximate method of dealing with these cases, but the characteristic performance may be calculated easily from the Langmuir $\frac{3}{2}$ -power law, and it is surprising that this has not been done. This law is applicable to most valves for all currents up to the normal saturation current of the cathode except for the very smallest values. Beyond the normal saturation current the conditions are too complex for any law to be applicable and it is possible that these may be the conditions contemplated by Shearing.

The methods adopted in the present Paper are similar to those of the 1919 Paper, and the performance of the rectifier is expressed in terms of the ratio of the steady voltage on the output side (V_0) to the peak value of the alternating input voltage (\hat{v}). The present Paper is, in fact, merely an extension of the earlier one.

THE MEAN CURRENT.

If \hat{v}_a is the instantaneous voltage between the anode and filament, and i is the instantaneous current, then

$$i = k \hat{v}_k, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is the expression of the $\frac{3}{2}$ -power law. In the present case the instantaneous current through one rectifying valve will be

$$i=k(\hat{v} \sin \theta - V_0)^{1/2},$$

where $v = \hat{v} \sin \theta$ is the applied alternating voltage, it being assumed that the output

* Fortescue, Proc. Phys. Soc., Vol. 31, Pt. 5, p. 319.

† Shearing, *Journal Inst. El. Eng.*, Vol. 63, p. 309 (1925).

voltage, V_0 , is sensibly constant owing to the necessity of using smoothing devices. (See Fig. 1.) The mean value of this current over the whole cycle is

$$I_0 = \frac{1}{\pi} \int_{\sin^{-1} \frac{V_0}{\hat{v}}}^{\frac{\pi}{2}} k(\hat{v} \sin \theta - V_0)^{\frac{3}{2}} d\theta$$

$$= \frac{k \hat{v}^{\frac{3}{2}}}{\pi} A_1, \quad \dots \dots \dots (2)$$

where

$$A_1 = \int_{\sin^{-1} \frac{V_0}{\hat{v}}}^{\frac{\pi}{2}} \left(\sin \theta - \frac{V_0}{\hat{v}} \right)^{\frac{3}{2}} d\theta$$

and can be evaluated.

From equation (2)

$$k \frac{V_0^{\frac{3}{2}}}{I_0} = \frac{\pi}{A_1} \left(\frac{V_0}{\hat{v}} \right)^{\frac{3}{2}} \quad \dots \dots \dots (3)$$

and if V_0 , I_0 , and a particular valve having a particular value of k are given, then only one value of $\frac{V_0}{\hat{v}}$ will satisfy equation (3). It is convenient to plot the values of

$\frac{\pi}{A_1} \left(\frac{V_0}{\hat{v}} \right)^{\frac{3}{2}}$ in the form of a curve, as in Fig. 2.

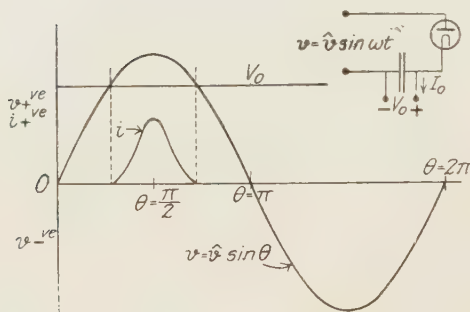


FIG. 1.

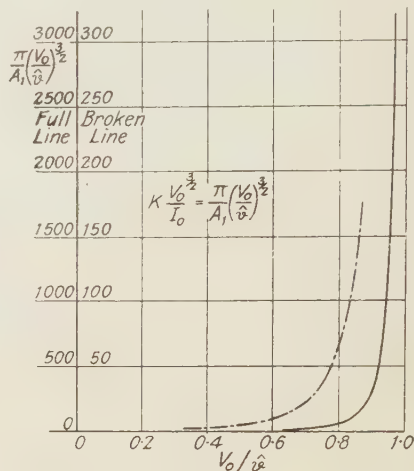


FIG. 2.

For any particular value of V_0/\hat{v} the maximum value attained by the current is

$$i_{\max.} = k(\hat{v} - V_0)^{\frac{3}{2}}.$$

The normal total emission current from the filament should be at least equal to this.

Putting

$$I_e = i_{\max.} = k(\hat{v} - V_0)^{\frac{3}{2}},$$

it follows from (2) that

$$\frac{I_e}{I_0} = \frac{\pi}{A_1} \left(1 - \frac{V_0}{\hat{v}} \right)^{\frac{3}{2}}$$

This ratio, again, may be plotted as in Fig. 3. It is noticeable that with high values of V_0/\hat{v} , the necessary emission current is many times more than $2I_0$.

If I is the R.M.S. current, it follows from (5) that

$$I = k\hat{v}^{\frac{3}{2}} \sqrt{\frac{A_3}{\pi}}$$

whence

$$\frac{I}{I_0} = \frac{\sqrt{\pi A_3}}{A_1}$$

This ratio is plotted in Fig. 3.

THE OVERALL EFFICIENCY.

A consideration of the curves of Fig. 3 shows that in practice there must be a compromise between efficiency and emission current. The power expended in the filament for the production of the emission current varies widely with the different conditions and is determined by the length of life that it may be considered advisable to provide for. For filaments of the same diameter the watts necessary per ampere of emission current increase with the length of life to be given. With varying

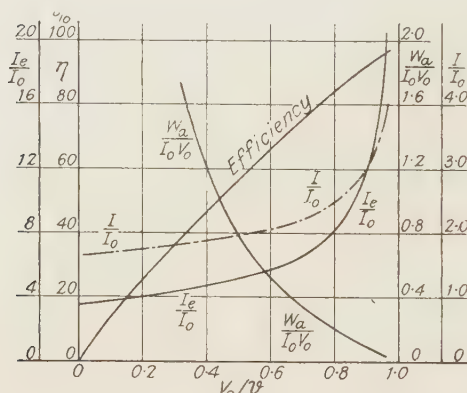


FIG. 3.

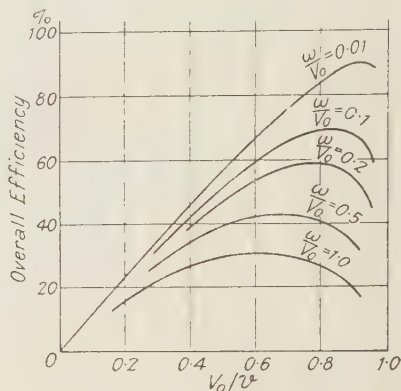


FIG. 4.

diameters of filament the same general conditions are applicable, viz., that the longer the life the larger the filament power for a given emission current. Assuming, therefore, that the watts per ampere of emission current are a reasonable basis of comparison it becomes possible to calculate the overall efficiency of the rectifier and so to get some guidance as to the best compromise between efficiency and emission current. If w denotes the watts per ampere of emission current then the filament power is wI_e and the overall efficiency is from equation (4)

$$\eta' = \frac{I_0 V_0}{k A_2 \hat{v}^{\frac{3}{2}} + w I_e}$$

Expressing the denominator in terms of I_0 and V_0

$$\eta' = \frac{A_1}{A_2 \frac{\hat{v}}{V_0} + \frac{w}{V_0} \left(1 - \frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}}$$

The values of η' may be plotted in terms of V_0/v for various ratios of w/V_0 as shown in Fig. 4.

ECONOMIC CONSIDERATIONS.

Further progress with the design of a valve for a given output depends upon a knowledge of the way in which the life of the valve depends upon the value of w . Assuming that a curve is available connecting the life in hours, H , and w ; then if D is the cost in pence of a valve for any particular purpose and d is the cost per unit of the power supplied, the cost in pence per hour of this valve during its working life is

$$\frac{D}{H} + \frac{I_0 V_0}{\eta'} \cdot \frac{d}{1000},$$

the values of H and η' being those corresponding to particular values of w . This quantity may be worked out and the minimum value found. This minimum gives the most economical valve provided that the value of k demanded by equation (3) can be satisfied. Unfortunately no very complete data connecting H and w are available.

It seems probable from such information as is available that for small low voltage rectifying sets the valves should be designed for long life and low overall efficiency. The ratio of η' may be less than 0.5 and a relatively low value of V_0/\hat{v} will be desirable. For instance, for an output of 10 milliamps at 200 volts from a single valve, the maximum current from the cathode would be in the neighbourhood of 50 milliamps and the voltage available to set up this current would probably exceed 200 volts. Such a valve would cost about £1, and if the cost of power is 1s. per unit, the life should be from 6,000 to 10,000 hours. The filament watts should be about 14 in a bright emitter valve. If a dull emitting filament is employed, with a corresponding reduction of the filament watts, a still longer life should probably be provided for.

DISCUSSION.

Dr. E. H. RAYNER said that it would be interesting to have data applicable to the design of rectifiers working at 4 to 6 volts, in addition to those for the 200-volt rectifiers considered in the Paper. He thought that in estimating the most economical design the life of a valve should be put at considerably less than the 6,000 to 10,000 hours assumed, since the mortality of valves due to accident is much greater than that due to wear of the filament.

Mr. R. P. FUGE said that the estimate would also be affected by the greater cost of using batteries instead of the mains for the filament supply, in accordance with a common practice.

The AUTHOR, in reply to the discussion, said that he had no statistics available as to the mortality of valves due to accidents. He had assumed that transformers would be used for the filament supply, as batteries were unnecessary.

XXVI.—THE THEORY OF LUMINESCENCE IN RADIOACTIVE LUMINOUS COMPOUND.*

By J. W. T. WALSH, *M.A., M.Sc., F.Inst.P.**Received December 20, 1926.*

ABSTRACT.

Previous work on this subject is reviewed, and the results of brightness measurements on zinc sulphide compounds containing radium are given for periods up to 4,000 days. From a comparison of the brightness curves of compounds made with the same luminescent material, but with different radium concentrations it is shown that the brightness-time relationship is of the form $B = rf(\nu t)$, where ν is the radium content.

The experimental facts available for developing and testing a theory of the luminescence are summarised, and a number of different theories, including a modified form of the recovery theory, are considered.

Finally it is shown that the observed brightness curves are in excellent agreement with Rutherford's original theory of the destruction of active centres, provided this be combined with a simple hypothesis as to the cause of the progressive increase in the light absorption of the material which has been found experimentally. This leads to the following form of the brightness-time relationship—

$$\log \{B/(b+B)\} + kt + a = 0,$$

where a , b and k are constants, of which the last two are proportional to the radioactive concentration for any given grade of luminescent material.

From the results obtained on two grades of compound it is concluded that (a) the rate of destruction of active centres is six to nine times that of the ionisation of inactive molecules, and (b) in the new material used for the compounds measured, 20 to 50 per cent. of the molecules are in the active state.

INTRODUCTION.

THE luminosity exhibited by certain specially prepared chemical compounds, when mixed with any radioactive material emitting alpha-particles at a constant rate, has been found to diminish gradually with lapse of time. The curve of decrease of brightness was first studied by Marsden in the case of willemite, zinc sulphide, and barium platinocyanide.† Marsden found, further, that—

(a) The brightness and not the number of the individual scintillations diminished with lapse of time.

(b) The luminosity was only slightly affected by change of temperature or by exposure to infra-red radiation.

(c) The luminosity was principally due to the alpha-particles, and only very slightly to the beta and gamma-rays.

His brightness curves were satisfactorily explained by Rutherford‡ on the assumption that the luminosity was due to the collision of the alpha-particles with certain "active centres" in the material, each collision giving a flash of light, and resulting in the destruction of the active centre.

Luminous compound consisting of a specially prepared zinc sulphide mixed with a radium salt was much used during the War, and measurements were made at the

* Thesis approved for the Degree of Doctor of Science in the University of London.

† E. Marsden, "The Phosphorescence Produced by the Alpha- and Beta-Rays," *Proc. Roy. Soc.*, 83, p. 548 (1910).

‡ E. Rutherford, "Theory of the Luminosity Produced in Certain Substances by Alpha-Rays," *Proc. Roy. Soc.*, 83, p. 561 (1910).

National Physical Laboratory on a number of samples of compound, some of which, by the kindness of the manufacturers* have been retained for measurement ever since they were made in 1915.

The results of measurements extending over a period of some 450 days were given in a previous Paper,† which also contained a description of the method employed for making the measurements of brightness and of radium content.‡

It was found that the simple exponential brightness curve resulting from the application of Rutherford's theory did not fit the observed results for more than a very limited period of time, as the percentage rate of decrease of the brightness gradually diminished. This led to the formulation§ of the "recovery" theory, in which it was assumed that the active centres were not finally destroyed after collision, but were restored at a rate proportional to their concentration. This theory was found to fit the observations then available and was therefore adopted as a working hypothesis to be subjected to the test of a more prolonged series of measurements when opportunity offered.

Since that time measurements have been made at intervals, particularly on Samples 1 and 2 of the original Papers (*loc. cit.* Notes † §), so that now curves covering a period of about 4,000 days are available. The object of the present Paper is to show the failure of the recovery theory to represent these long-period curves, and by a discussion of them and of other results obtained more recently, to obtain a more satisfactory theory of the decay of luminosity.

THE EXPERIMENTAL DATA: CORRECTION FOR GLASS ABSORPTION.

Previous work on the subject has always, of necessity, been based on comparatively short period brightness curves (usually one year or less), so that this is the first occasion on which it has been possible to make a thorough test of any proposed formula. Long period curves are indispensable for making such a test, since, partly on account of the uneven distribution of the radium emanation through the bulk of the mixture, and partly on account of the difficulties of photometry at such low intensities, the accuracy of the measurements made on a single occasion is probably not greater than about 10 per cent. Figures read from a smoothed curve should, however, be correct to about 5 per cent., except at the very low values which have to be measured when the compound is old. The order of accuracy may be gauged from Figs. 3 and 4, in which the actual readings are shown by the points.

There is a further source of error which cannot be eliminated or even reduced by smoothing. This is the progressive coloration of the glass walls of the vessel used for containing the compound, and a consequent continual increase in the light absorption. This must be allowed for in the final brightness curves.

* Mr. F. Harrison Glew and Messrs. W. Watson & Sons (Electro-Medical), Ltd.

† C. C. Paterson, J. W. T. Walsh and W. F. Higgins, "An Investigation of Radium Luminous Compound," *Proc. Phys. Soc.*, 29, p. 217 (1917); *N.P.L. Collected Researches*, Vol. 15, p. 289 (1920).

‡ The same methods have been used in the present work, but the brightness unit has been changed to the candle per square metre. The results given in the previous Papers referred to above were expressed in "equivalent foot-candles"—i.e., the brightness of a perfectly diffusing surface of 100 per cent. reflection factor having an illumination of 1 foot-candle. One equivalent foot-candle equals 3.425 candles per square metre.

§ J. W. T. Walsh, "The Theory of Decay in Radioactive Luminous Compounds," *Proc. Roy. Soc.*, 93, p. 550 (1917); *N.P.L. Collected Researches*, Vol. 15, p. 313 (1920).

It is probably safe to assume that this effect is due to the destruction of certain molecules within the glass* at a rate proportional to the radioactive concentration. If these molecules absorb light in accordance with Beer's law, the transmission factor τ follows an exponential law with respect to both time and radioactivity.

Although τ is difficult to measure accurately, and is, moreover, not uniform over the walls, measurements have been made in the case of two of the original samples containing respectively 200 and 100 micrograms of radium element per gram. The measured transmissions at the end of 3,900 days were respectively 70 and 84 per cent., after allowing for losses by reflection at the glass surfaces. Since $(\log 0.70)/(\log 0.84)$ is very nearly equal to 2 it follows that, to the necessary degree of approximation, it is sufficient to regard the transmission factor as exponential with respect

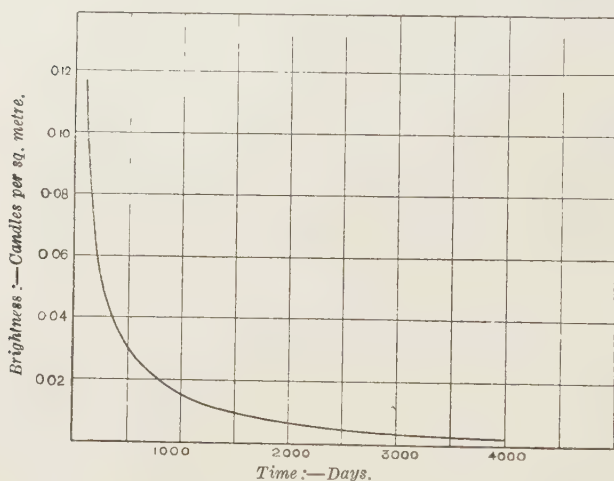


FIG. 1.—OBSERVED BRIGHTNESS CURVE FOR SAMPLE 1 (0.2 MG. Ra/GM.).

to the radium content, and therefore also with respect to the time since $\tau = f(rt)$. The same conclusion was arrived at in the case of the other samples for which it was possible to make measurements of τ . The expression $\tau = e^{-pt}$ has, therefore, been used throughout this Paper for correcting the observed values of brightness, the value of p adopted for each sample being that deduced from measurements on the tube containing that sample, or on a tube made from the same specimen of glass.

RELATION BETWEEN THE BRIGHTNESS CURVES FOR SAMPLES HAVING DIFFERENT RADIOACTIVE CONCENTRATIONS.

From what has been said above with regard to the accuracy of the observations it will be clear that even the long period brightness curves are, by themselves, insufficient to provide a really rigid test of any theory that may be proposed. It is therefore fortunate that a further and more certain test can be obtained by comparing the brightness curves of different samples, made up from the same batch of luminescent material, but with different proportions of the radioactive constituent. Such a comparison may be made in the case of Samples 1 and 2 of the original investigation, which contain respectively 200 and 100 micrograms of radium element per gram, the luminescent material being the same for both. The corrected brightness curves of these samples are shown in Figs. 1 and 2.

The circles in Fig. 2 indicate the course of the curve of Fig. 1 when its abscissæ have been multiplied, and its ordinates divided by 2, i.e., in the ratio of the radium

* It should be mentioned that the coloration may be completely destroyed by heating the glass in a Bunsen flame.

contents of the samples. The difference between the two is everywhere very small, and is, in fact, generally less than the experimental error, so that it would appear that the brightness curves for a set of samples made up with a given luminescent material may be represented by the equation $B_t = r f(rt)$ where r is the radium content and t the time.

This conclusion is of such importance, that it is desirable to confirm it by means of samples made with other zinc sulphides, and, where possible, by results published by other experimenters. It is therefore fortunate that an opportunity has lately been presented for making such a confirmation in connection with an investigation of different grades of zinc sulphide which has recently been carried out at the National Physical Laboratory.* Six pairs of samples were made in the Chemical Department of the Laboratory, the samples in each pair being made from the same consignment of zinc sulphide, but with radium contents in the ratio of approximately

2.5 to 1. Measurements on some of these samples have been continued for over 1,200 days.

The samples were all mixed by the "wet method," i.e., the zinc sulphide was mixed with the desired quantity of radium bromide in the form of an aqueous solution just sufficient in volume to moisten the whole of the sulphide, and the superfluous moisture was then evaporated. The samples were hermetically sealed in glass tubes of 8 mm. diameter immediately after mixing, and their radium contents were de-

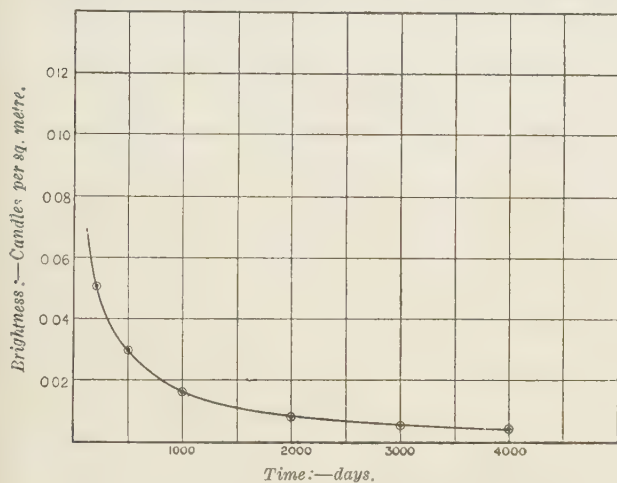


FIG. 2.—OBSERVED BRIGHTNESS CURVE FOR SAMPLE 2 (0.1 MG. Ra/GM.), WITH VALUES CALCULATED FROM FIG. 1 (CIRCLES).

termined in the Radiology Division of the Laboratory using the gamma-ray method. Allowance was made for the absorption of the gamma-rays in the substance of the compound.†

The confirmation obtained with one pair of these samples is shown in Fig. 3, where the full-line represents the brightness curve observed in the case of Sample A, containing 193 micrograms of radium per gram, while the circles indicate points obtained from the smoothed curve for a second Sample, H, made with the same zinc sulphide, but containing 552 micrograms of radium per gram (Fig. 4). The positions of these circles have been calculated from the points marked in Fig. 4, by dividing the ordinates and multiplying the abscissæ by the ratio 552/193. The

* This was carried out for the Chemistry Research Board of the Department of Scientific and Industrial Research, and the writer's thanks are due to the Board for their permission to make use of the results obtained in the course of the investigation.

† E. A. Owen and Winifred E. Fage, "The Estimation of the Radium Content of Radioactive Luminous Compounds," *Proc. Phys. Soc.*, Vol. 23, p. 27 (1921).

other pairs of samples made at the same time from other grades of zinc sulphide show a similar agreement.

In a report of the U.S. National Advisory Committee for Aeronautics* brightness curves, extending over a period of about 260 days, are given for compounds containing respectively, 100, 150, 220, and 300 micrograms of radium element per gram of compound. The relationship already noted is strikingly confirmed in the case of these samples, the agreement being generally within about 3 per cent.

The results given in the previous Paper by the writer and others† also confirm this conclusion although the curves as published only extend over about 500 days.

The only published observations which appear to contradict this relationship are those given by G. Berndt for a number of samples made with the same zinc sulphide, but with radium contents varying from 0.02 to 0.21 mg. radium element per gram.‡ Berndt, however, took as the starting point of his time scale the time

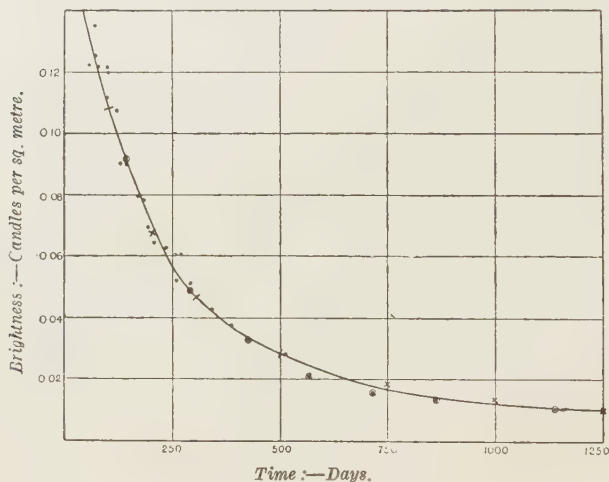


FIG. 3.—SAMPLE A.

Observed Brightness Curve (Full line).

Values Calculated from Fig. 4 (Circles).

Values Calculated on Light Absorption Theory (Crosses).

Observed Values are Indicated by Points.

brightness curve of a sample containing any given amount of radium can be obtained in, say, one third or even one quarter of the full time by measuring a sample containing three or four times the amount of radium per gram.

of maximum brightness and not the actual time of mixing, and if the origin of the time scale be shifted back 20 days so as to allow for this, Berndt's results are found to be in excellent agreement with the relationship $B/r=f(rt)$. The agreement is, in fact, generally better than that obtained by Berndt for his proposed formula. These observations, therefore, afford further confirmation of this important relationship.

It may be mentioned in passing, that there is an important practical application of this principle in the life-testing of luminescent zinc sulphides. For, clearly, the

* Report No. 33, "Self-Luminous Materials," by N. E. Dorsey. It may be noted in passing that a purely empirical expression for the brightness curve was obtained at the Bureau of Standards, viz., $B_t=(a+bt)^{-1}$, where a and b were constants (see N. E. Dorsey, Proc. Washington Acad. Sci., 7, p. 1 (1917), and Trans. Am. Electrochem. Soc., 32, p. 389 (1917), in discussion). No attempt has apparently been made to establish this formula on a theoretical basis, and it fails, in fact, to represent the results over more than a very limited period of time.

† Paterson, Walsh and Higgins, loc. cit., p. 237.

‡ G. Berndt, "Der Helligkeitsabfall radioaktiver Leuchtfarben," Z. f. techn. Physik., Vol. 1, p. 102 (1920).

OTHER OBSERVED FACTS.

In addition to the relationship noted above, the facts observed by Marsden (*loc. cit.*) as well as the following have to be considered when testing any theory of luminosity :—

- (1) Pure zinc sulphide is not radio-luminescent. Active zinc sulphide is crystalline and in the samples used for the present work, the average linear dimension is of the order of 4×10^{-4} cm., i.e., about one tenth of the range of an alpha-particle in ZnS. Crushing the crystals in a mortar, either before or after mixing with the radium, causes a marked reduction in the brightness.
- (2) There are two principal bands in the spectrum of the light emitted by a zinc sulphide compound. The one which contributes most to the luminosity

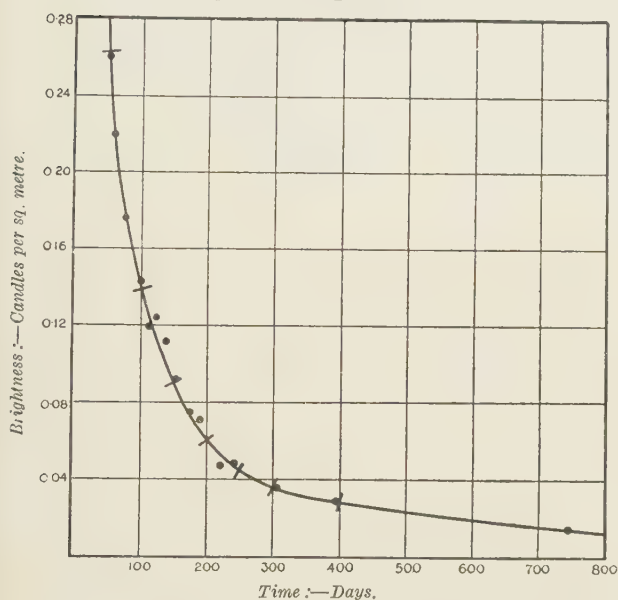


FIG. 4.—SAMPLE H. OBSERVED BRIGHTNESS CURVE WITH ACTUAL OBSERVED VALUES INDICATED BY POINTS.

kept always in the dark.

- (4) The observed curves show that (a) B_t tends to zero as t increases, and (b) after about 700 to 800 days the brightness of a sample containing 100 micrograms of radium per gram is definitely brighter than that of a sample containing twice this amount or more.

THEORIES OF THE LUMINOSITY : (a) THEORIES LEADING TO A DOUBLE EXPONENTIAL.

The most natural form of expression after a simple exponential is the compound exponential, $B_t = \sum_{i=1}^n B_i e^{-k_i t}$. This is of the form $rf(rt)$, as long as B and k are propor-

* See G. Berndt, "Radioaktive Leuchtfarben," (Vieweg, 1920), p. 91 and Figs. 22 and 24. Marsden, *loc. cit.*, p. 553. H. Herszfeld and L. Wertenstein, "Phosphorescence du Sulfure de Zinc sous l'action des Rayons Alpha," J. de Phys., 2, p. 31 (1921).

osity has its maximum in the neighbourhood of 550 $m\mu$. The other is much less luminous and considerably broader. Its maximum is in the neighbourhood of 460 $m\mu$. It is the diminution of brightness of the band of longer wave-length which is responsible, at any rate in the main, for the decay of the luminescence of the compound.*

- (3) The photo-luminescence (response to light) of zinc sulphide mixed with radium gradually diminishes even though the sample be

tional to r . In order to test the agreement of a formula of this type with the observed brightness curves a method was developed for analysing any curve into its component exponentials.* Since each component has two parameters the observed curves are not definite enough to test any hypothesis leading to more than two exponentials. Two such hypotheses at once suggest themselves: (i.) The active centres may be of two kinds, each having its own characteristic rate of decay; or (ii.) the active centres, originally all of one kind, may be transformed, after the impact of an alpha-particle and the emission of light, into a second variety of active centre, which, if struck by another alpha-particle, will emit a second flash of light. It is clear that the second hypothesis leads to an expression for B_t exactly analogous to that obtained for the activity curve of a pair of successive radioactive elements.†

The only direct evidence which can be brought forward to support either of these theories is the progressive change in the relative intensities of the two bands in the spectrum of the emitted light (*see* above), which might very naturally be expected to result from a gradual change in the relative proportions of two kinds of active centres, or from a gradual substitution of "secondary" for "primary" active centres.

The earlier portion of the brightness curve of Sample 1 can be represented satisfactorily by the double exponential‡ $B_t = 0.1335 \text{ antilog}_{10}(-0.00326t) + 0.0474 \text{ antilog}_{10}(-0.000525t)$.

This expression, calculated from the values of brightness at 100, 300, 500 and 700 days, gives the broken line curve of Fig. 5, while the actual brightness curve is shown in full line. It will be seen that the theoretical curve is not really of the shape required to represent the observations. No doubt a sufficiently good agreement could be obtained with a triple exponential, but as this would contain six independent parameters, the adoption of a theory based on such an expression would appear to be highly artificial.

THEORIES OF THE LUMINOSITY: (b) THE THEORY OF SURFACE ACTION.

Berndt found§ that his curves were well represented by $B_t = B_0(1 - e^{-At})/At$, where A and B_0 are constants. This is the formula given by Rutherford's theory on the assumption that the radium is not distributed uniformly through the material, but is confined to the surface of a sheet or screen;|| it might, for instance, be considered as deposited on the external surfaces of the zinc sulphide crystals.

A formula of this type fits the curves fairly well for a considerable part of the time, as may be seen from Fig. 5, where the circles represent the formula

$$(14.23/t) \{1 - \text{antilog}_{10}(-0.004687t)\}.$$

The values at 150 and 300 days were used to determine the constants. The formula

* J. W. T. Walsh, "The Resolution of a Curve into a Number of Exponential Components," *Proc. Phys. Soc.*, 32, p. 26 (1919); *see* also H. Levy, *Proc. Phys. Soc.*, 34, p. 108 (1921).

† E. Rutherford, "Radioactive Substances and their Radiations," p. 421 (1913); H. Bateman, "The Solution of a System of Differential Equations Occurring in the Theory of Radioactive Transformations," *Proc. Camb. Phil. Soc.*, 15, p. 423 (1910).

‡ The common antilogarithm has been used instead of the exponential in order to facilitate calculation. The above expression is equivalent to $0.1335e^{-0.0075t} + 0.0474e^{-0.00121t}$.

§ G. Berndt, "Der Helligkeitsabfall radioaktiver Leuchtfarben," *Z. f. techn. Physik.*, Vol. 1, p. 102 (1920); "Radioaktive Leuchtfarben," pp. 86 to 90.

|| This was, in fact, the case actually investigated by Marsden (*loc. cit.*).

fails progressively after about 2,000 days, however, and this is only to be expected, since it is not in agreement with the observed fact that after long periods of time a sample of lower radium content has a higher luminosity. It will be seen that according to this formula the sample of higher content has always the higher luminosity.

Further, as has been mentioned, the linear dimensions of the crystals are very much less than the range of the alpha-particle, so that a true surface effect can only be postulated on the assumption that the power of the alpha-particle to destroy an active centre decreases much more rapidly along its path than its ionizing power in the case of a gas.

THEORIES OF LUMINOSITY: (c) THE MODIFIED RECOVERY THEORY.

The simple recovery theory fails to represent the long period brightness curves, and the same is true of a modification proposed by Witmer,* who assumed that the rate of recovery was affected by the radiation emitted within the material. Witmer's equation for B was $\log(B - B_\infty) - \log(B + B') = -kt + l$. He found this was in good

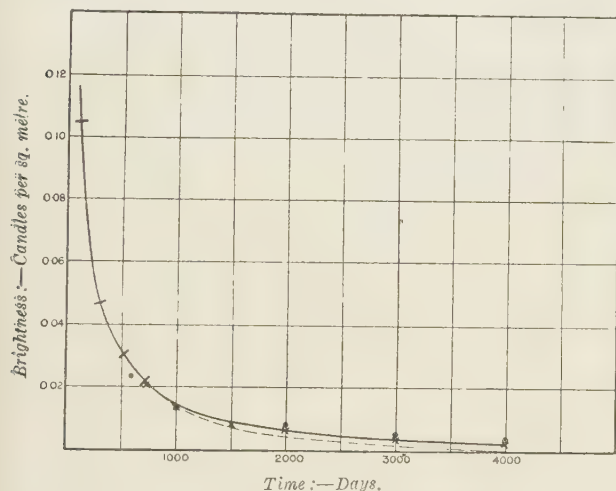


FIG. 5.—SAMPLE I.

Observed Brightness Curve (Full line).
Double Exponential Curve (Broken line).
Surface Action Theory (Circles).
Light Absorption Theory (Crosses).

agreement with the brightness curve obtained by Rodman for pure radium bromide, and for a mixture of radium and barium bromides.† Rodman's radioactive concentrations were, however, extremely high, and it is not surprising to find that the same form of equation does not represent the results on ordinary zinc sulphide compounds. It leads, for instance, to a value of B_∞ which is greater than zero.

This objection may be overcome by assuming that the recovery of the active centres is entirely due to the emitted radiation. This assumption is in accordance with the well-known hypo-

thesis that the speed of molecular reactions is dependent on the flood of radiation of a particular frequency which pervades the reacting system. If the energy emitted when an active centre is destroyed is, as seems reasonable on this theory, of the frequency necessary to produce recovery, and if it be assumed that the rate of recombination is proportional to the number of destroyed centres

* E. E. Witmer, "The Theory of Alpha Ray Luminescence," *Phys. Rev.*, Vol. 24, p. 639 (1924).

† J. A. Rodman, "The Effect of Temperature on the Luminosity of Radium Compounds," *Phys. Rev.*, 23, p. 478 (1924).

and to the intensity of the radiation, the equation giving the nett rate of decrease of active centres is

$$\frac{dn}{dt} = -k_1 n + k_2(n_0 - n) \cdot k_1 n,$$

where n is the number of active centres present at time t and n_0 is the number present initially. The brightness, being proportional to the nett energy emission, is given by

$$B = -m(dn/dt).$$

These equations give for the brightness-time relationship

$$be^{ct} = \frac{\sqrt{1+aB}-1}{\sqrt{1+aB}+1}$$

where a , b and c are independent constants of positive sign. This relation between B and t fails to represent the observed brightness curve of Sample 1 with any real positive value of a even with the values of B_{100} , B_{300} and B_{500} changed by amounts which are quite outside the experimental error.

THEORIES OF THE LUMINOSITY : (d) THE THEORY OF SECONDARY IONISATION.

In discussing Marsden's results Rutherford showed that the active centres were destroyed at about 15 times the rate of the inactive molecules. He suggested that this might be due either to greater cross-section or to an instability such that destruction might be caused by the secondary radiation set up by the alpha-particle in the molecules it bombarded. This hypothesis does not change the form of the equation for B , but such a change is brought about if it be supposed that the destruction of one active centre affects others in its neighbourhood, and, in fact, leads to their destruction at a rate proportional to their concentration in the material.

On this hypothesis the aggregate rate of destruction

$$\begin{aligned} -dn/dt &= k_1 n + k_1 n \cdot k_2 n + k_1 k_2 n^2 \cdot k_2 n + \dots \\ &= k_1 n / (1 - k_2 n), \end{aligned}$$

where k_1 and k_2 are constants, of which k_1 is proportional to the radioactive content.

This gives, on integration,

$$\log_e (n/n_0) - k_2(n - n_0) + k_1 t = 0,$$

and, since $B = -m(dn/dt)$, $n = B / (k_1 m + k_2 B)$.

Hence, $\log \{B / (k_1 m + k_2 B)\} - k_2 B / (k_1 m + k_2 B) + k_1 t = \text{a constant}$.

This may, clearly, be rewritten thus :—

$$\log B / (b + B) - B / (b + B) + at + c = 0,$$

where

$$a \equiv k_1, \quad b \equiv m k_1 / k_2,$$

and c is given by the condition that $B = B_0$, when $t = 0$.

It is clear that a and b are essentially positive, and proportional to the radioactive concentration, so that the equation is of the form $B = rf(rt)$.

Also $B_\infty = 0$.

The relation, in fact, represents the observed brightness curves to a very reasonable degree of accuracy, and the chief objection to it is the high rate of secondary ionisation which it is necessary to assume during the early part of the life. In the case of Sample 1, for instance, the best values of the constants are $a=7.2 \times 10^{-4}$, $b=0.051$, and $c=1.000$. From these values it is easy to deduce that even after the lapse of 100 days, each destruction of an active centre leads to the destruction of a second centre in about four cases out of every five.

An alternative hypothesis which leads to an exactly similar form of brightness curve is that the radiation emitted as a result of the destruction of active centres by bombardment causes the formation of active centres from inactive centres, and that the radiation emitted as a result of this recovery action results, in its turn, in a further destruction of active centres, and so on, it being assumed that the number of inactive centres is so great that it may be regarded as sensibly constant.

THEORIES OF THE LUMINOSITY: (e) THE LIGHT ABSORPTION THEORY.

The above brief outline of some of the more promising theories which have been tested will serve to show the many different directions in which an explanation of the observed phenomena has been sought. With the exception of the two alternatives described in the immediately preceding section, none of the theories gives an expression which is in satisfactory accord with the observed brightness curve.

The chief objection to both the theories outlined in Section (d) above is the somewhat artificial nature of the hypotheses upon which they are based. In consequence they have now been discarded in favour of a much simpler and more natural theory, which leads to an expression of very similar form and which gives at least as good, if not better, agreement with the observed brightness curve. It gives, in addition a simple explanation of the decay of photo-luminescence in a radio-active compound (see § (3), p. 323), a fact which had not previously been accounted for.

Several determinations have been made at different times* of the transmission of the zinc sulphide itself for the light emitted as a result of the radioactive bombardment. If the absorption factor α be defined by the equation $B_p = Be^{-\alpha p}$ where B_p/B is the fraction of light transmitted by a layer of the material of thickness p cm.,† then α ranges from about 30 to 100 cm.⁻¹ in the case of new compound. It has been found by Clinton,‡ however, that α increases rapidly with lapse of time. In the case of the compound he used, for which the radium content was about 150 micrograms per gram, the value of α at 0, 100 and 200 days was 32, 46 and 52 cm.⁻¹ respectively. This rapid increase in the absorption factor of the material itself seems to have escaped notice as a possible cause of the rapid initial rate of decrease of the brightness. It will be shown in the next section, however, that it gives, when applied to Rutherford's simple decay theory, a brightness curve which is in excellent agreement with observation.

It may be noted in passing that, as mentioned above, this effect explains very simply the gradual decrease of photo-luminescence in a radioactive compound, for the penetration of the incident light to the interior of the salt, and the emission

* F. Bahr, "Die Oekonomie der radioaktiven Leuchtfarben," Zeits. f. Beleuchtungswesen, Vol. 22, p. 153 (1916). G. Berndt, "Radioaktive Leuchtfarben," p. 66.

† See, e.g. J. W. T. Walsh, "Photometry," p. 116 (London, 1926).

‡ W. C. Clinton, "Some Photometric Tests of the Brightness of Radioactive Self-Luminous Materials," Illum. Eng., Vol. 11, p. 260 (1918).

of the light produced by the photo-luminescent action, are both rapidly reduced by the increase in the absorption factor of the material.

EXPERIMENTAL PROOF OF THE LIGHT ABSORPTION THEORY.

It is clear that if the absorption factor of the luminescent material at any time be α , and if the total light flux emitted from an elementary volume $d\sigma$ of the material be $Fd\sigma$ lumens, the candle-power of the surface due to this element of volume will be $(Fd\sigma/4\pi)e^{-\alpha x}$, where x is the distance of the element behind the surface*. Thus the candle-power per unit area, due to a layer of thickness dx at a distance x behind the surface, is $(Fdx/4\pi)e^{-\alpha x}$. It follows that the total brightness of a sheet of material of thickness x is $(F/4\pi)\int_0^x e^{-\alpha x}dx = (F/4\pi\alpha)(1 - e^{-\alpha x})$.

Now if it be supposed that the ionization of both the active and inactive molecules of the material results in an increase of α by an amount which is proportional to the concentration of ionized molecules in the material (Beer's law), it is clear that instead of α there must be written an expression of the form $\{\alpha + \beta(n_0 - n) + \gamma t\}$ where $(n_0 - n)$ is the number of destroyed active centres, and α , β , γ are constants. Although the third term should, strictly, be of the same form as the second, the rate of decrease in the number of the inactive centres is so slow that the rate of destruction may be regarded as sensibly constant. It follows that the brightness of an infinitely thick layer of material is given by $B = F/4\pi\{\alpha + \beta(n_0 - n) + \gamma t\}$ or, writing $F/4\pi = -m(dn/dt)$

$$B\{\alpha + \beta(n_0 - n) + \gamma t\} = -m(dn/dt)$$

Rutherford's simple destruction theory gives $n = n_0 e^{-kt}$,

so that

$$dn/dt = -kn_0 e^{-kt}$$

and, finally,

$$B\{\alpha + \beta n_0(1 - e^{-kt}) + \gamma t\} = mkn_0 e^{-kt}$$

or

$$\log_e \frac{B(1+ct)}{b+B} + a + kt = 0$$

where

$$a \equiv \log(1 + \alpha/\beta n_0)$$

$$b \equiv mk/\beta$$

and

$$c \equiv \gamma/(\alpha + \beta n_0)$$

It will be seen that this expression for the brightness curve contains four constants, viz., a , b , c and k . Since m , α and β are, *a priori*, independent of the radioactive content, while k and γ are proportional to it, it follows that b and c are also proportional to this quantity, and that the equation is of the form $B = r f(rt)$, while $B \propto 0$.

Further, it is clear that a and k are essentially positive on the basis of the original assumptions, while a , b and c can also be shown to be positive on *a priori* reasoning. For B decreases to zero as a limit so that, since $\log B(1+ct)/(b+B)$ must always be real, if b is negative, c is also negative, and, further, $ct = -1$ when $B = -b$. Moreover under these conditions β is negative, but it is clear that βn_0 must be numerically

* See, e.g. J. W. T. Walsh, "Photometry," pp. 86 and 116.

less than a since the absorption factor can never be less than zero. Hence for c to be negative, γ must be negative.

Now it is easy to show (vide infra) that the rate of ionization of the inactive molecules is about 0.0069 per cent. per day, so that if γ is negative as well as β , a must, as before, be numerically greater than $\beta n_0 + \gamma / 0.69 \times 10^{-4}$ and therefore c must be numerically less than 0.69×10^{-4} . It follows that when $ct = -1$, t is greater than 14,500, so that B_t and therefore b is so small as to be negligible for at least the first 500 days. This is not in accordance with the observed form of the brightness curve, so that it must be concluded that both b and c are positive in sign. Since a and β are thus both positive, so also is a .

It is found that the expression

$$\log \frac{B(1+ct)}{b+B} + a + kt = 0$$

represents the observed brightness curve of sample l to a very satisfactory degree of accuracy, when c is put equal to zero, and the fit of the curve is not improved by adopting a finite positive value for c . It follows that the expression for the brightness curve of this sample of compound may be written

$$\log \frac{B}{b+B} + a + kt = 0$$

The values of the constants calculated from the observed values of B at 100, 300 and 500 days are as follows :—

$$a = 0.030$$

$$b = 0.0077$$

$$k = 4.08 \times 10^{-4}$$

Exactly similar expressions are found to fit the observed brightness curves of all the samples studied. The constants for Sample A, for instance, are as follows :—

$$a = 0.043$$

$$b = 0.012$$

$$k = 6.15 \times 10^{-4}$$

The degree of approximation with which this expression represents the observations in these two cases, is shown in Figs. 5 and 3 respectively, where the full line curves are the observed brightness curves, while the theoretical values deduced from the algebraic expression are indicated by crosses. It will be seen that the agreement is everywhere exceedingly good, and well within the limit of 5 per cent. mentioned in the early part of the Paper. This fact, combined with the simplicity of the underlying assumptions, gives the light absorption theory considerable value as a working hypothesis of the mechanism producing the luminosity in a radioactive compound. It will therefore be of interest to make some deductions which, if the necessary assumptions be valid, give interesting information as to the stability and concentration of the active centres in a luminescent zinc sulphide.

THE RATE OF DESTRUCTION OF ACTIVE CENTRES.

If it be assumed that the number of molecules of zinc sulphide ionized by an alpha-particle is the same as the number of ions produced by the same alpha-particle in a gas,* it is possible to deduce at once the relative rates of destruction of the active and inactive molecules. For in one gram of compound containing 200 micrograms of radium element, the number of alpha-particles emitted per second from the radium and its products as far as Radium C is $4 \times 3.4 \times 10^{10} \times 2 \times 10^{-4}$, i.e., the number per day is 2.35×10^{12} . The number of ions produced in the same time is, therefore, 4.4×10^{17} . The total number of molecules in a gram of zinc sulphide is 6.38×10^{21} . The rate of destruction of inactive centres is, therefore, 0.69 per cent. per 100 days.

Now the rate of destruction of the active centres is clearly equal to k , i.e., in the case of Sample 1 to 4.1 per cent. per 100 days. It must, therefore, be concluded that for this sample of compound the active centres are about six times as liable as the inactive molecules to destruction by the bombardment of the alpha-particles. For Sample A the ratio is about nine. It is interesting to compare these figures with that of 15 obtained by Rutherford (*vide supra*), who did not allow for the effect of light absorption.

THE CONCENTRATION OF THE ACTIVE CENTRES.

Another interesting estimate which can be made from the values of the constants in the expression for the brightness curve is that of the value of n , i.e., the number of active centres present in the luminescent material. For if it be assumed that each active centre emits, on destruction, one quantum of energy in the form of radiation of wavelength $550 m\mu$, the mean wavelength of the emission band of a zinc sulphide compound,† the energy emitted per second is $\frac{-dn}{dt \times 24 \times 3,600} \times 3.6 \times 10^{-12}$ ergs, i.e., the luminous flux emitted is $(-dn/dt) \times 660 \times 3.6 \times 10^{-19} \div 86,400$ lumens, since the mechanical equivalent of light of wavelength $550 m\mu$ is about 660 lumens per watt.‡ But the luminous flux emitted in one cubic centimetre of the compound is clearly $4\pi B\{a + \beta n_0(1 - e^{-kt})\} \times 10^{-4}$, if B is expressed in candles per square metre and a and β are in cm.^{-1} . Also, since $a = \log(1 + a/\beta n_0)$ this expression may be written

$$4\pi B a \left\{ 1 + \frac{1 - e^{-kt}}{e^a - 1} \right\} \times 10^{-4} \text{ lumens.}$$

Equating these two results, the number of active centres destroyed per day in one cubic centimetre of the material is found to be

$$\frac{4\pi B a (e^a - e^{-kt}) \times 8.64}{(e^a - 1) \times 660 \times 3.6 \times 10^{-19}}$$

i.e., the number present at time t is

$$\frac{\beta a (e^a - e^{-kt})}{k(e^a - 1)} \times 4.57 \times 10^{17}.$$

* Rutherford, loc. cit., p. 571.

† See e.g., E. Marsden, loc. cit., p. 553; H. Herszfeld, L. Wertenstein, "Phosphorescence du Sulfure de Zinc sous l'action des Rayons Alpha," J. de Phys., Vol. 2, p. 31 (1921); G. Berndt, "Radioaktive Leuchtfarben," pp. 90-92.

‡ See e.g., J. W. T. Walsh, "Photometry," p. 465.

The most interesting value of n is n_0 , i.e., the number of active centres present in the luminescent material before any radioactive bombardment has taken place. Owing to the fact that the radioactive bombardment is not constant, but gradually rises to its equilibrium value during the first 30 days after mixing, the simple form of the brightness curve given above cannot be used for this calculation, but the special form of the curve appropriate to the initial part of the life (for the first 100 days) must be determined.

THE INITIAL PART OF THE BRIGHTNESS CURVE.

It has been shown in a previous Paper* that, during the first thirty days after mixing, the alpha-ray activity of the material at any time t may be taken as $r(1 - \frac{4}{5}e^{-\lambda t})$, where r is the equilibrium activity, and $\lambda = 0.180 \text{ day}^{-1}$. It follows that

$dn/dt = -nk(1 - \frac{4}{5}e^{-\lambda t})$, or $\log n/n_0 = -kt + \frac{4k}{5\lambda}(1 - e^{-\lambda t})$. From this result it is easy

to show that the formula for the brightness curve is $\log \frac{B}{b' + B} + a' + kt = 0$,

where

$$a' \equiv a + \frac{4k}{5\lambda}e^{-\lambda t},$$

$$b' \equiv b(1 - \frac{4}{5}e^{-\lambda t}),$$

and a , b and k have the same values as before.

$$\text{Thus,} \quad \log \frac{5B_0}{b + 5B_0} + a + 4.44k = 0,$$

which gives for Sample 1, $B_0 = 0.048$ and for Sample A, $B_0 = 0.051$. It will be clear that the expression for n_0 becomes, similarly, $(5B_0\alpha/k) \times 4.57 \times 10^{17}$, or, putting $\alpha = 50$, $(B_0/k) \times 1.14 \times 10^{20}$. In the case of Sample 1 the value of n_0 is 1.3×10^{22} , so that if the density of the material be put equal to 4, the number of active centres originally present per gram is 3.2×10^{21} , i.e., about 50 per cent. of the total number of molecules. The corresponding figure for Sample A is 37 per cent. In considering the probability of these figures it has to be remembered that they are directly proportional to the value assumed for the absorption factor of the zinc sulphide before bombardment. Clinton's results gave a figure of 32 for this quantity, instead of the figure of 50 assumed in the above calculation.

In view of the large uncertainty introduced by this factor, no correction has been made for the fact that the values of brightness obtained were as measured through the wall of a glass tube. They should, therefore, be increased by about 10 per cent. on this account.

Although little reliance can be placed on the actual values of n_0 obtained it is satisfactory to find that they are of a reasonable order of magnitude.

In the case of Sample A actual measurements of brightness were made from within a few hours of the actual time of mixing. It is interesting to compare the

* J. W. T. Walsh, loc. cit., Note § (p. 319).

observed with the calculated brightness curve for the period 0 to 100 days. This has been done in Fig. 6, where the full line represents the calculated brightness curve, using the values of the constants given on p. 12 above. The points represent the observations, and it will be seen that while the theoretical value of B_0 is in excellent agreement with the measured value, the subsequent observations all lie below the theoretical curve for the first 70 to 80 days.

Exactly the same thing has been found in the case of the other samples for which observations are available for the initial period, although the difference between the observed and calculated values is not always as large as that in the case of Sample A. The explanation of the discrepancy seems to be that there is a diffusion of radium emanation over the whole tube, so that while the initial values of brightness are in agreement, the subsequent values are lower than would be expected until a state of "equilibrium" has been established.* There is no doubt whatever that emanation

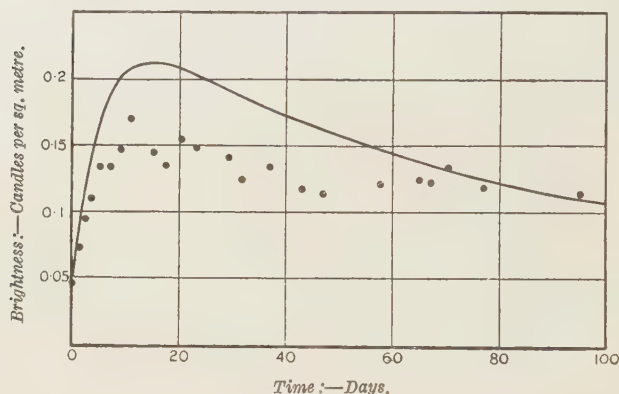


FIG. 6.—INITIAL PORTION OF THE BRIGHTNESS CURVE* OF SAMPLE A. THEORETICAL CURVE (FULL LINE) WITH OBSERVED POINTS.

escapes from the compound since tubes which were not efficiently sealed at first were found to give consistently low values of gamma-ray activity until re-sealed. The fact that the observed values do not lie on a smooth curve (to the accuracy of measurement) indicates that the escape of the emanation from the compound is, to a certain extent, irregular.

CONCLUSIONS.

From the results given in the Paper, and the confirmation afforded by the other samples of compound studied, it may be concluded that the mechanism of the luminosity production is of the kind originally postulated by Rutherford to explain Marsden's results. In other words, the light emission is the result of the destruction by alpha-particles of certain "active centres" in the luminescent material. The rate of destruction is proportional to the radioactive concentration and to the concentration of active centres in the material. In a compound of constant activity, therefore, the rate of decrease of the active centres follows an exponential curve, and so does the amount of light actually emitted in unit volume of the material.

As time goes on, the transparency of the luminescent material gradually decreases, so that the curve of brightness of the material in bulk is not exponential, but is of the form represented by the equation $\log B/(b+B) + kt + a = 0$, where B is the brightness at time t , and a , b and k are constants. The values of b and k are, for a given luminescent material, proportional to the radioactive concentration, while a is independent of this quantity. It follows that, as found experimentally, the brightness curve of a compound of radioactive concentration r may be obtained

* See, in this connection, E. Rutherford, "Radioactive Substances," pp. 361 *et seq.*

from that of a compound of concentration r' made up with the same luminescent material by multiplying the ordinates (B) and dividing the abscissæ (t) of the latter curve by the factor r/r' .

The brightness curves obtained by the application of this theory are in excellent agreement with those found experimentally, except during the first 70 to 80 days after mixing, when the observed values are always lower than those obtained by the modified formula applicable to the initial part of the brightness curve of a radium compound mixed by the wet method (during the attainment of radioactive equilibrium). This difference may be attributed to the diffusion of radium emanation over the interior of the container.

On the assumption that the number of molecules of zinc sulphide ionized by an alpha-particle is the same as the number of ions produced by such a particle in air, it follows that the rate of destruction of active centres is about six to nine times that of ionization of the inactive molecules.

On the assumption that each active centre emits, on destruction, one quantum of energy in the region of the spectrum which corresponds with the maximum of the principal luminous band of a zinc sulphide compound, it can be deduced that about 25 to 50 per cent. of the molecules are in the active state in the case of new material such as that used for the compounds studied.

The writer's acknowledgments are due to Dr. E. A. Owen, lately in charge of the Radiology Division of the National Physical Laboratory, and to Mr. T. E. Rooney, of the Chemistry Department, who were associated with him in the investigation referred to on p. 321. He also wishes to thank Mr. B. J. W. Oram, of the Photometry Division, who co-operated in the somewhat arduous photometric work involved in the determination of the brightness curves.

DISCUSSION.

LORD RAYLEIGH pointed out that since the radio-active material was mixed with the luminescent material throughout the experiment, it had been impossible to test the effect of rest on the latter of these materials. He was glad that the author had succeeded in disentangling the facts in spite of this difficulty, but he would like to suggest that future investigations might be simplified by separating the constituents of the luminous compound. He had recently been using the natural luminosity of certain uranium salts as a photometric standard in the study of the light coming from the sky at night. Uranium nitrate, for instance, is luminescent in response to light or to the action of radio-active substances, and consequently is faintly self-luminous in response to the uranium which it contains. It would be interesting to subject it to the influence of radium emanation or some similar substance, and to find out whether its luminescence can be fatigued. Although the luminosity of uranium salts is much fainter than that of the compound discussed in the Paper, it is practically permanent, and its marked difference in this respect might be worth investigating.

Dr. H. BURNS : Can the author say anything further about the influence of purposely added impurities, to which he has only referred in his introduction ?

Dr. D. OWEN asked whether anything was known as to the nature of the "active centres." Did the spectrum of the emitted light give any information on this subject ?

The AUTHOR, in reply, to the discussion, said that the separation of the constituents of the compound as suggested by Lord Rayleigh would certainly afford the right method for a more fundamental investigation than that described in the Paper, but the latter had been concerned primarily with a practical problem, and had, therefore, been directed to the compound in the form in which it is usually employed. The luminosity of the compound could be made more permanent while its intensity was diminished by reducing the radium content, and a sufficient reduction might conceivably lead to properties comparable with those of a uranium salt. The nature of the impurities to which Dr. Burns had referred was to some extent a trade secret. The report of the Chemical Research Board mentioned in the footnote on p. 4 of the Paper was a confidential document. Sir Ernest Rutherford had suggested that the "active centres" might be complex molecules, but the author did not himself hold any theory as to their nature.

XXVII.—DISTORTION OF RESONANCE CURVES OF ELECTRICALLY-DRIVEN TUNING FORKS.

By E. MALLET, D.Sc., M.I.E.E., A.M.I.C.E.

Received December 10, 1926.

ABSTRACT.

In the first section of the Paper the experimental arrangements employed to obtain accurate resonance curves are described. In the second section resonance curves with increasing exciting currents show increasing distortion until an unstable state of affairs is arrived at in which the amplitude for a given current over a certain frequency range can have two different values, depending upon whether the frequency has been approached from above or below. The indication here is that a decrease of resonant frequency takes place with increase of amplitude, and it further appears that an increase of damping also takes place. Experiments with a free fork showed that these effects were still present, though not to the same extent as with the driven fork. Static experiments showed a departure from the straight line law both in the case of the deflection of the fork prongs for given loads, and the flux change through the core for a given deflection of the prongs. In section III the effect of such departures on the equation of motion is considered mathematically; it is shown that the term depending on the cube of the amplitude is the most important and that its inclusion gives rise to resonance curves of the form observed. The effect of the static square term is shown to be in effect a dynamic cube term. In section IV the experimentally obtained resonance curves are examined in the light of the theory, and substantial agreement is found, while in section V a detailed calculation of the various constants is made. In section VI another effect of the non-linearity is found in the possibility of producing fundamental frequency vibrations in the fork by means of exciting currents of double frequency, and the effect of a first harmonic in an exciting current of fundamental frequency is considered.

In section VII distortions of a second type are shown to consist of "coupled circuit" effects: these at large amplitudes are modified by distortions of the first type.

CONTENTS.

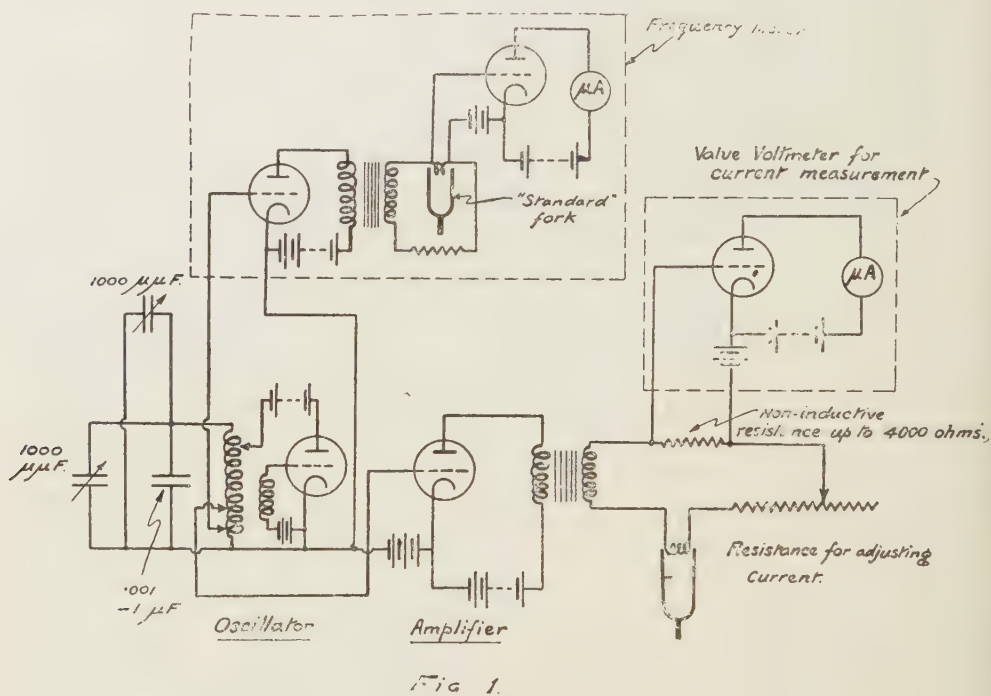
- I. Experimental Arrangements.
- II. First Experiments.
 - A. Resonance Curves.
 - B. Free Fork Experiments :—
 - (i) Frequency studied by Lissajous Figures.
 - (ii) Damping studied by Photographic Decay Curves.
 - C. Static Experiments :—
 - (i) Mechanical Stiffness of Fork.
 - (ii) Flux alterations with Deflection.
- III. Theoretical.
 - Curves are drawn on the assumption of an additional restoring force and an additional damping term varying as x^3 , and found to be similar to the experimentally obtained curves.
 - Static x^2 terms lead to dynamic x^3 terms.
- IV. Application of Theory to Experimental Results.
- V. Calculation of Fork Constants.
- VI. Double Frequency Excitation.
- VII. Distortions of Second Type—coupled circuit effects.
- Conclusion.

prongs was the same under the conditions described. When the amplitude was small an attempt was made to measure it by means of a Kennelly mirror, but this was not successful as will be seen.

II. FIRST EXPERIMENTS.

A. Resonance Curves.

Fig. 2 shows a typical resonance curve obtained on the Koenig fork *M13*. The core of the exciting coil was $1\frac{1}{4}$ in. below the tips of the prongs. The double amplitude as measured on the microscope, is plotted against the oscillator capacity in μFs . The amplitude was measured some distance (2.64 cms.) below the tip of the prongs. One thousand divisions on the microscope correspond to 1 mm. actual deflection,



or an amplitude of vibration of $\frac{1}{2}$ mm. To find the amplitude at the tip in mms. the deflections must be multiplied by 1.19, or the actual microscope readings by 0.000595.

The frequency differences are found by multiplying δC in microfarads by 314. The current was kept constant at 1.51 mA.

At first following Kennelly on the receiver diaphragm distortion* it was thought that the shape of this resonance curve was due to some mechanical "coupled circuit" effect. Then it was observed that with larger exciting currents and consequently larger amplitudes, under certain conditions without any circuit alterations the amplitude gradually increased until the fork prongs were striking the core.

* Kennelly, "Electrical Vibration Instruments," Chap. XII.

To elucidate this effect curves were drawn of amplitude against current at different frequencies. Two of these are reproduced in Fig. 3, in which curve (a) was taken at a frequency less than resonance (i.e., less than 320), and curve (b) at a frequency greater than resonance.

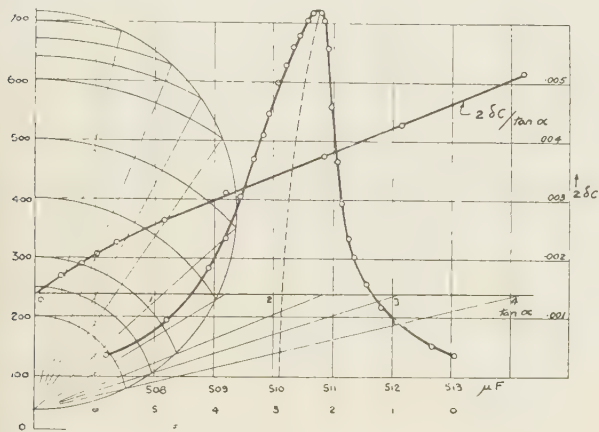


FIG. 2

It is seen that in the latter case the increase of amplitude with current is less as the latter increases, but in the former it is greater and greater, and finally an unstable position is reached at which the amplitude goes on increasing until the fork strikes the core.

These results can be explained if it be assumed that the resonant frequency of the fork decreases as the amplitude increases; so that the point at any amplitude

mid-way between the two sides of the resonance curve of Fig. 2 gives the capacity value corresponding to the resonant frequency of the fork at that amplitude. Then in curve (b) of Fig. 3, as the current increases and the amplitude increases the resonant frequency decreases and so the exciting frequency is still lower than the resonant frequency and the amplitude increase is not so large as it would be were there no change of resonant frequency. But in curve (a) where the exciting frequency is below the resonant frequency of small amplitude, an increase of current and amplitude brings the resonant frequency closer to the exciting frequency, and so the amplitude increase is greater than would be accounted for by the increase of current alone.

The instability would be explained by the change of impedance of the fork as its resonant frequency changes, and the resulting change in current with constant voltage from the oscillator. For instance, let *OP*, Fig. 4, represent the impedance at a particular frequency and amplitude, and *AR* the resonant motional impedance. Then an increase of amplitude resulting in

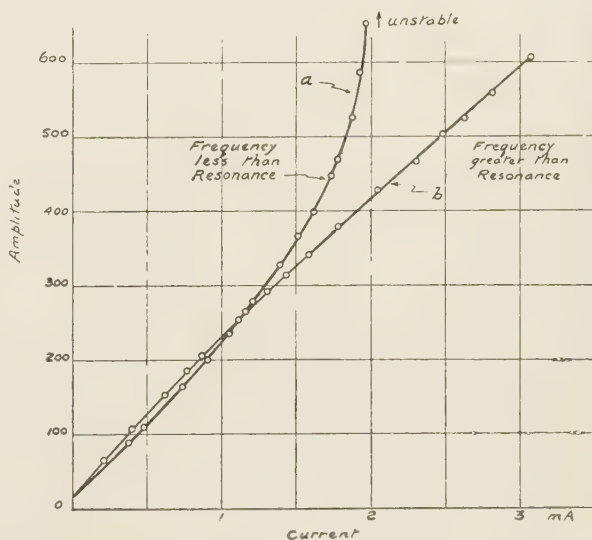


FIG. 3.

a decrease of natural frequency will move the point P round towards R , say, to P' . Thus the impedance OP is reduced to OP' with a corresponding increase of current and amplitude, and hence a still further rotation of P round the circle. For this reason when the constant current curves such as Fig. 2 were taken, it was found necessary to have a large resistance in series with the fork, and to have a correspondingly powerful amplifier valve.

Another unstable effect was observed with a sufficiently large constant current, and taking care that the current was never increased above the fixed value by suitable manipulation of the series resistances. When the frequency was gradually decreased from a high value, the amplitude rose to a certain value and then after a slight decrease, suddenly decreased by a large amount to a stable value, and then finally gradually decreased. On increasing the frequency again the amplitude retraced the first curve and went beyond the point of sudden decrease before there was a sudden increase, and a final retracing of the initial curve. The process is indicated by the firm lines and arrows in Fig. 5. It was as though the resonance curve had been completely turned over at the tip, but that the dotted part was unstable and incapable of being traversed.

Similar results have been obtained by Prof. E. V. Appleton* with a vibration galvanometer, but the Author was not aware of this until the present Paper was in print.

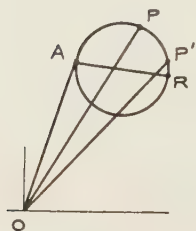


Fig. 4.

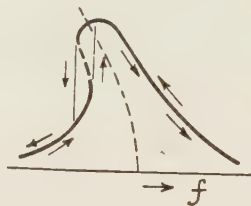


Fig. 5.

To settle finally that there actually was a change of resonant frequency with amplitude a series of peaks was drawn with different exciting currents. This family of curves is shown in Fig. 6, where it is seen that the capacity at which the peak occurs increases progressively as the current is increased;

that is to say, there is a marked decrease of resonant frequency with amplitude.

Another interesting point is brought out in Fig. 6 by plotting the maximum amplitude against the current. It is seen that this curve is concave towards the abscissa, thus indicating that the damping increases with increase of amplitude.

This increase of damping with amplitude is also indicated by carrying out the circle and straight line construction† on the resonance curve of Fig. 2. In order to eliminate the change of resonant frequency the construction is modified slightly by plotting the distance from one side of the resonance curve to the other against $\tan \alpha$ instead of as usual the distance from one side to the resonant frequency. Since the decay factor is proportional to the slope of the curve $\delta C / \tan \alpha$ obtained, it is clear that there is an increase of damping at the higher amplitudes.

B. Free Fork Experiments.

It was next desired to find out whether these effects were due to the fact that the fork was driven in the manner described, or whether they were present when the fork was vibrating freely without the presence of the bobbin and coil.

* Phil. Mag., Vol. 47, p. 609.

† J.I.E.E. 62, p. 523, and Expl. Wireless, February, p. 95 (1927).

(i) The fork was clamped so that it could be set in vibration by bowing. Another electrically-driven fork with adjustable riders was set up at right angles so that Lissajous figures were obtained on a screen, using a pointolite lamp and a suitable optical arrangement with small plane mirrors on the prongs of the forks. The driven fork was vibrated by the oscillator first at a slightly lower frequency and then at a slightly higher frequency than the natural frequency of the fork, maximum amplitude being obtained in each case by adjusting the position of the riders. The free fork was vigorously bowed. The times at which the Lissajous figure was a straight line, that is, the half beats, were recorded on a telegraphic "local inker" by tapping a key. The distance between the marks obtained was proportional to the time of a half-beat. The intervals were somewhat erratic, but plotting the mean

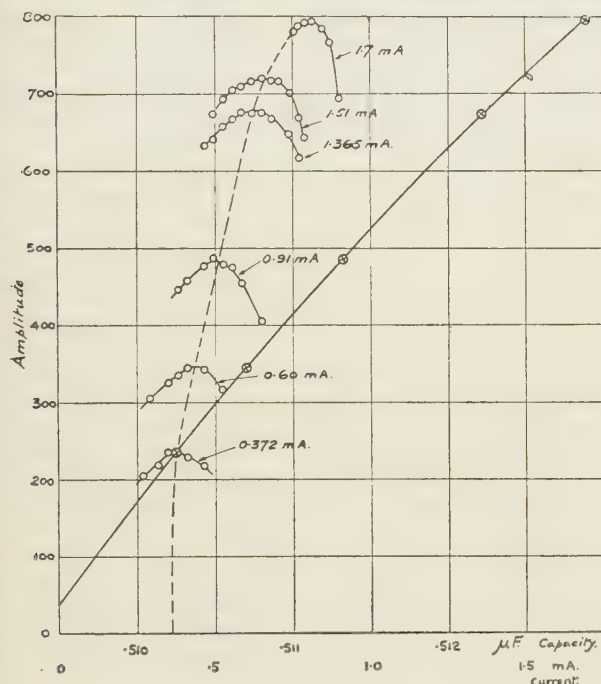


Fig. 6.

of each 4 intervals against the time, curves were obtained which gave quite definite indications with each setting of the driven fork. In the first case the time of the beat definitely increased, showing that as the amplitude decreased the frequency of the free fork departed further from that of the driven fork; that is, it increased. In the second the beat time decreased; the frequency of the free fork approached more closely that of the driven fork—it increased.

The change of frequency as calculated from these curves was of the order of 2 to 3 parts in 10,000, whereas the change in Fig. 10 is about 8 parts in 10,000. The largest amplitude in the two

cases was roughly the same. The conclusion is, therefore, that there is a frequency change with amplitude in the free fork. There is further an indication that the frequency change in the driven fork is made up of two parts; one inherent in the fork itself and the other due to the driving arrangements.

(ii) The damping of the free fork was investigated by taking decay curves photographically. Two such curves were studied in this connection, (a) obtained with the electrical excitation by breaking the oscillator circuit, and (b) obtained with the bobbin removed.

Since the amplitude after any time, t , is $x = x_0 e^{-\Delta t}$, $\Delta = -2.3 \frac{d \log x}{dt}$, for any amplitude is to be found as 2.3 times the slope of the curve obtained by plotting the logarithm of the amplitude against the time.

In this way the curves of Fig. 7 were obtained from the photographic records. In each case there is an increase of damping with amplitude, and not only the initial damping but also the rate of increase of damping with amplitude is greater with the electrical driving arrangements in position than when they are removed.

C. Static Experiments.

(i) Since part at any rate of the change of frequency with amplitude occurs with the free fork, it would be expected that a non-linear curve would be obtained by a static test of deflection for various loads.

The fork was accordingly clamped rigidly by its handle to the bed plate of a drilling machine, and loads were applied by placing weights in scale pans attached by cords passing over suitable pulleys to stirrups on the prongs. In order that the load could be applied both to draw the prongs closer together and to separate them, one stirrup was made longer than the other, and had holes drilled in it through which the cords from the shorter stirrup passed. Deflections were read by the same microscope used previously. To avoid moving the microscope to read the deflection of each prong, a short strip of copper was attached to the end of each prong and

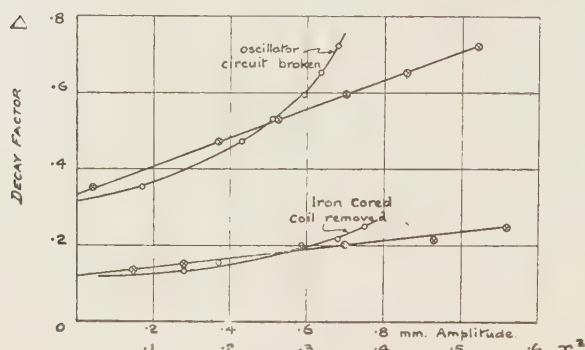


Fig. 7.

marks made on each which were visible at the same time in the microscope field.

A distinct lack of symmetry was found in the curve of deflection/load, but sufficient accuracy to evaluate the constants in an expression of the form $f = Au + Bu^2 + Cu^3$ was not attained. It was clear, however, that some such expression would be necessary.

(ii) In the simple linear theory of the telephone receiver, which can be applied with very slight modifications to the driven tuning fork, the driving force is shown to be proportional to the permanent flux, which is taken to vary linearly with the deflection. The flux produced by the current is also taken to be a linear function of the air gap.

The next experiments were directed towards examining the validity of these assumptions. The terminals of the coil of a fork were connected to a galvanometer, and the ballistic deflection observed when the coil and core together snatched away from their position between the prongs of the fork. This was repeated with various air gaps, obtained by clamping the prongs with a brass clamp. The ballistic deflections are proportional to the flux Φ_0 through the core due to the permanent magnetism of the fork.

Next a special 50 turn coil was wound on the bobbin and a current started through this with the core in position between the prongs. The ballistic deflections of the galvanometer were now proportional to the flux Φ_i produced by the current for different air gaps.

In each case there is a marked departure from a straight line. For deflections

of 2 mm. on each side of the mean position, the flux can be represented by expressions of the form $\Phi = A + Bx + Cx^2 + Dx^3$, and the values of $\frac{B}{A}$, $\frac{C}{A}$ and $\frac{D}{A}$ were nearly the same for Φ_0 and Φ_i .

III. THEORETICAL.

In the simple theory of the fork, calling M the magnetic motive force of the permanent magnetism, and i the current through the coils (in absolute units), x the displacement from mean position towards the core, R the reluctance in the mean position, S the cross-sectional area of air gap, B_0 the flux density due to the permanent magnet, and N the number of turns on the core, the magnetic pull on each prong = $\left[\frac{B^2 S}{8\pi} \right]$

$$= \left\{ \frac{M + 4\pi Ni}{S \left(R - \frac{2x}{S} \right)} \right\}^2 \cdot \frac{S}{8\pi}$$

and when x is small so that $\frac{2x}{SR} \ll 1$,

The pull =

$$\frac{\Phi_0^2}{8\pi S} + \frac{B_0^2 x}{2\pi R} + \frac{NB_0 i}{R} + \dots \text{ terms in } i^2 \text{ and } i^2 x.$$

The first term is the steady pull, the second a pull depending on the position of the prong, and the third the alternating pull. The remaining terms are small, involve harmonic distortion and are neglected. Thus the equation of motion of the prong can be written:—

$$m\ddot{x} + r\dot{x} + sx = Ai + \frac{B_0}{\pi R}x$$

or

$$m\ddot{x} + r\dot{x} + s_1 x = Ai$$

where $s_1 = \left(s - \frac{B_0}{\pi R} \right)$, i.e., the effect of the positional pull is to modify the stiffness of the prong.

Actually owing to eddy currents and hysteresis, there will be a lag of the flux behind the current and of the positional force behind x , so that A will be complex, and the positional force will involve also an increase of the effective mechanical resistance r . But these changes will not alter the simple linear relation between x and i .

Now when the amplitude is large, it has been seen that statically at any rate, the flux in the air gap can be written $(\Phi_0 + \Phi_i)(1 + bx + cx^2 + dx^3)$, and the pull r is proportional to the total flux squared, or very nearly

$$(\Phi_0^2 + 2\Phi_0\Phi_i + \Phi_i^2)(1 + 2bx + 2cx^2 + 2dx^3) \dots \dots \dots (2)$$

This expanded gives rise to a number of terms, the interest in which chiefly lies in

those containing Φ_i , x , x^3 , $\Phi_i x^2$, and $\Phi_i^2 x$, Φ_i gives the driving pull, and x the positional pull of the linear theory, while the remainder, since both x and Φ_i are varying very nearly sinusoidally, contain terms which also vary sinusoidally with the same frequency. We may in fact assume, at any rate, to a first approximation, that the motion of the fork is simple harmonic, and analyse $x^3 \sin^3 \omega t$, $\Phi_i \cdot \sin \omega t \cdot x^2 \sin^2 \omega t$, and $\Phi_i^2 \sin^2 \omega t \cdot x \sin \omega t$ trigonometrically into terms of fundamental frequency, and triple frequency, and neglect all but the terms of fundamental frequency in finding an equation for the motion.

In this way the amplitude of the total force actuating the fork may be written

$$A \Phi_i (\cos \varphi + j \sin \varphi) + Bx (\cos \alpha - j \sin \alpha) + Cx^3 (\cos \beta - j \sin \beta) \\ + D \Phi_i x^2 (\cos \gamma - j \sin \gamma) + E \Phi_i^2 x (\cos \delta - j \sin \delta) \dots \dots \dots (3)$$

where the phase of the motion x is reckoned as standard, where φ is the angle of lead of the current in front of the motion, less the angle of lag of the flux behind the current, and the angles $\alpha, \beta, \gamma, \delta$ represent the lags of the various forces, produced, as in the simple theory, by the eddy current and magnetic hysteresis losses. Of these forces the first is the main driving force, the second and the last are simple positional forces, which can be taken as modifying r and s , while the third and the fourth can produce distortion of the resonance curve. Of these, the fourth, except at small amplitudes of very large currents, is small compared with the third, and will be neglected at first to simplify the theory. There is left therefore a force

$$A \Phi_i (\cos \varphi + j \sin \varphi) + \frac{3}{4} C x^3 (\cos \beta - j \sin \beta) \text{ of fundamental frequency.}$$

On the other side of the equation of motion it has been seen that the restoring force is not linear for large amplitudes, but can be written $sx - s_1 x^2 - s_2 x^3$. Now if x is varying sinusoidally, the x^2 term will introduce no fundamental frequency restoring force, but the x^3 term will. Also if there is mechanical hysteresis, this restoring force will lag behind x , and must be written in the equation

$$\frac{3}{4} s_2 x^3 (\cos \xi - j \sin \xi)$$

So that the equations are now

$$-m\omega^2 x + j\omega r x + sx - \frac{3}{4} s_2 x^3 (\cos \xi - j \sin \xi) \\ = A \Phi_i (\cos \varphi + j \sin \varphi) + \frac{3}{4} C x^3 (\cos \beta - j \sin \beta) \dots \dots \dots (4)$$

$$\text{or} \quad \left. \begin{aligned} \frac{-m\omega^2 + s}{\omega r} x - Bx^3 &= A \cos \varphi \dots \dots \dots \\ x + Dx^3 &= A \sin \varphi \dots \dots \dots \end{aligned} \right\} (5)$$

The constant D expresses the increase of resistance with amplitude, and B the decrease of frequency with amplitude, while the new constant A depends upon the current.

In order to find whether these equations can give curves similar to those obtained experimentally, including the unstable effects, D is neglected, as it will only involve a flattening of the curves more pronounced as the amplitude is increased, and a series of curves drawn.

Taking $B=0.5$ and $A=1, 1.5, 2$ and 2.5 , for various values of x , the corresponding values of ϕ are found for different values of $\frac{-m\omega^2+s}{\omega r}$. Curves (5) (i) and (ii) are then plotted to the same base ϕ , and the intersection of the two curves gives x and ϕ for the value of $\frac{-m\omega^2+s}{\omega r}$ and therefore of ω taken.

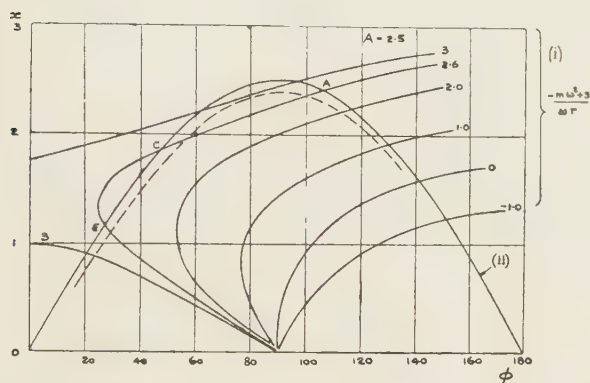


Fig. 8.

As an example the curves with $A=2.5$ are given in Fig. 8.

The solutions of x from these curves, and from similar curves drawn with $A=1, 1.5$ and 2 , are plotted in Fig. 9 against $\frac{-m\omega^2+s}{\omega r}$, which for small dampings as obtained with tuning forks is proportional to the frequency.

It is seen that the resonance curves of Fig. 9 are for the smaller amplitudes of the same shape as the experimental curve of Fig. 2, and that the peaks of the curve Fig. 6 also conform to the general arrangement. Further with still larger amplitudes, the tip of the curve actually does turn over as was expected from the unstable experimental result.

That the underside of the curve from B to D in Fig. 18 represents an unstable equilibrium can be readily seen from the constant frequency curves of Fig. 8. For any slight increase in the resistance, r will flatten curve (ii) say to the dotted line, resulting in a reduction of amplitude at points A and E , which is the natural result. But in the case of point C the theoretical conditions can only be met by an increase

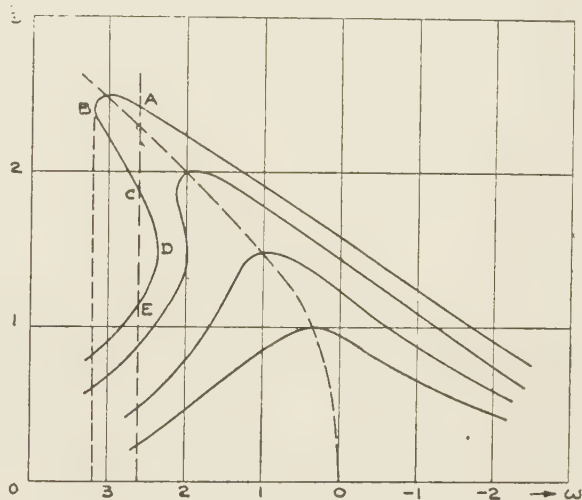


Fig. 9.

of amplitude, which there is no means of achieving. The amplitude therefore goes on falling to E , where conditions are again stable. On the other hand, a reduction of resistance at C necessitates a reduction of amplitude. This is impossible, and the amplitude goes on increasing until the point A is reached.

The theory developed can, therefore, in a general way, explain the experimental results. The changes of frequency given by the theory, agree with the expression given in Lord Rayleigh's "Sound," Vol. 1, p. 78, for the frequency reduction with amplitude of a freely vibrating fork without damping. Rayleigh's formula 7 is

$$m^2 = n^2 + \frac{3}{4}\beta A^2$$

Here n is the small amplitude value of 2π times the natural frequency of the fork, m is the value at an amplitude A , and β is the co-efficient of u^3 in the equation

$$\ddot{u} + nu + \beta u^3 = 0$$

for the free vibrations of the fork (undamped).

We started with an equation

$$m\ddot{x} + rx + sx + kx^3 = Ai$$

which on the assumption of x varying sinusoidally, and of free undamped vibrations, becomes

$$-m\omega^2 x - sx + \frac{3}{4}kx^3 = 0$$

so that $\frac{3}{4}k$ is what we have called s' , and $-\frac{k}{m}$ corresponds to Rayleigh's β .

Now the frequency at maximum amplitude is obtained, as will be seen from Fig. 8, when $\varphi = 90^\circ$, and then from (5) with $D=0$,

$$x = \frac{A}{\omega r} = A_0$$

where A_0 = the amplitude at resonance,

and

$$\ddot{m} \left(-\omega^2 + \frac{s}{m} \right) = s' A_0^2$$

Writing $\frac{s}{m} = \omega_0^2$, the small amplitude value (Rayleigh's n^2) this gives

$$\omega_0^2 - \omega^2 = \frac{s'}{m} A_0^2 = \frac{\frac{3}{4}k}{m} A_0^2 \quad (7)$$

whence

$$\omega^2 - \omega_0^2 = \frac{3}{4}\beta A_0^2$$

in agreement with Rayleigh's approximation.

The square terms, both in the actual restoring force on the fork, and in the magnetic forces acting, which can be looked upon as modifying the restoring force, have up till now, been neglected. They can be collected together into one term $s_1 x^2$ and will result in the restoring forces not being symmetrical about the position of rest. Thus the restoring force for a movement one way will be $sx - s_1 x^2$, and for a movement the other way $sx + s_1 x^2$. The result will probably be that the mean position of the prongs when vibrating will differ slightly from their rest position.

If we assume that at the extreme positions, to the right and to the left, the potential energies are equal, and that the movement from the rest position to the left is x_1 and to the right x_2 , we shall have

$$\int_0^{x_1} (sx + s_1 x^2) dx = \int_0^{x_2} (sx - s_1 x^2) dx$$

or

$$\frac{1}{2}sx_1^2 + \frac{1}{3}s_1x_1^3 = \frac{1}{2}sx_2^2 - \frac{1}{3}s_1x_2^3.$$

If still assuming that the fork is vibrating sinusoidally, we write for the vibration amplitude $x_m = \frac{x_1 + x_2}{2}$, and for the shift of the mean position $x_0 = \frac{x_2 - x_1}{2}$ we find that very nearly

$$x_0 = \frac{1}{3} \frac{s_1}{s} x_m^2$$

The restoring force term in the equation of motion becomes therefore, dropping the subscript m ,

$$\begin{aligned} & s(x + x_0) - s_1(x + x_0)^2 \\ &= sx - \frac{2}{3} s_1 x^2 - \frac{2}{3} \frac{s_1^2}{s} x^3 - \frac{1}{9} \frac{s_1^2}{s^2} x^4 \end{aligned}$$

The last term is small and is neglected. The third term is a cube term and can be dealt with as previously. We still have a second term in x^2 . Applying this again with $\frac{2}{3} s_1$ instead of s_1 , we shall have an additional cube term with co-efficient $\frac{2}{3} \left(\frac{2}{3} s_1 \right)^3 \cdot \frac{1}{s}$, and a remaining square term of co-efficient $\frac{2}{3} \cdot \frac{2}{3} s$, and so on. The final co-efficient of the cube term will be

$$-\frac{2}{3} \cdot \frac{s_1^2}{s} \left\{ 1 + \left(\frac{2}{3} \right)^2 + \left(\frac{2}{3} \right)^4 + \dots \right\} = -\frac{6}{5} \cdot \frac{s_1^2}{s}$$

Thus the equation of motion becomes

$$m\ddot{x} + r\dot{x} + sx - \frac{6}{5} \frac{s_1^2}{s} x^3 = A$$

and the previous investigation with the cube terms will cover the case of the square terms also.

For instance, the frequency change at maximum amplitude is found by writing

$$k = \frac{6}{5} \cdot \frac{s_1^2}{s}$$

in equation (7), giving

$$\begin{aligned} \omega_0^2 - \omega^2 &= \frac{3}{4} \cdot \frac{6}{5} \cdot \frac{s_1^2}{sm} A_0^2 \\ &= \frac{9}{10} \cdot \frac{s_1^2}{\omega_0^2 m^2} \cdot A_0^2 \end{aligned}$$

Rayleigh gives for the frequency change due to the square term

$$m^2 = n^2 - \frac{5a^2 A^2}{6n^2}$$

where a is defined by the equation

$$u + n^2 u + au^2 = 0$$

In our nomenclature, a is $\frac{s_1}{m}$, and using this Rayleigh's expression becomes

$$\omega^2 - \omega_0^2 = -\frac{5}{6} \frac{s_1^2}{\omega_1^2 m^2} A_0^2$$

It is seen that the correction is of the same order in each expression, but that our co-efficient is too large, being 0.9 as compared with Rayleigh's 0.833. The

frequency change introduced by the square term is in any case small compared with that due to the cube term, and it is evident that to a sufficiently close approximation, the cube term investigation should cover the experimental results.

IV. APPLICATION OF THEORY TO EXPERIMENTAL RESULTS.

Since the whole of the resonance takes place with only a small change of ω , it is possible to write $\frac{-m\omega^2 + s}{\omega r} = K\delta C$, where K is a constant and δC the change

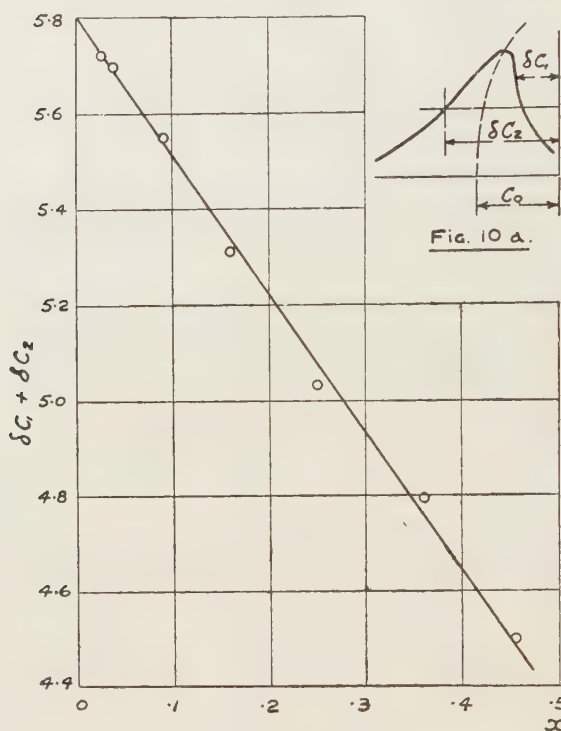


Fig. 10.

slope is $-\frac{2B}{K}$ and whose intercept on the axis of y gives $2C_0$.

The following table is drawn up from Fig. 6, taking the origin at $0.513 \mu F$, the unit of δC as $0.001 \mu F$, and a reading of 100 on the microscope as 0.1 , and allowing 40 divisions as zero error.

x	δC_1	δC_2	$\delta C_1 + \delta C_2$	x^2
0.16	1.00	4.75	5.72	0.025
0.20	1.35	4.35	5.70	0.040
0.30	1.77	3.78	5.55	0.090
0.40	1.91	3.40	5.31	0.160
0.50	2.02	3.10	5.03	0.250
0.60	2.08	2.72	4.80	0.360
0.675	2.25	2.25	4.50	0.455

$\delta C_1 + \delta C_2$ is plotted against x^2 in Fig. 10.

of oscillator capacity from the small amplitude resonance value. At equal values of x , the left hand side of equation (5) (ii) is the same and therefore $\sin \phi$ is the same. At these points (5) (i) gives the two equations:

$$\begin{cases} K\delta C_1 x - Bx^3 = A \cos \phi \\ K\delta C_2 x - Bx^3 = A \cos \phi \end{cases}$$

where $A \cos \phi$ is in the first positive and equal to $-A \cos \phi$ in the second.

Adding gives

$$K(\delta C_1 + \delta C_2)x - 2Bx^3 = 0 \quad (8)$$

The δC_2 may contain a constant if we do not know the small amplitude value of C at resonance. For instance, suppose the δC_2 measured from 0 where the capacity difference is C_0 to the resonance value (Fig. 10a).

Equation (8) gives

$$2C_0 - (\delta C_1 + \delta C_2) - 2Bx^3 = 0 \quad (9)$$

If, therefore, $\delta C_1 + \delta C_2$ is plotted against x^2 a straight line should result, whose

It is seen from the figure that the points lie quite well on a straight line, whose intercept on the y axis is $5.8=2C_0$ (from which $C_0=2.9$ fitting in well with the value indicated by Fig. 6) and whose slope $=2.92$, giving $\frac{B}{K}=1.46$.

Thus the frequency change checks up with the theory quite well. The damping is most conveniently examined from the family of peaks, Fig. 6. At the peaks $\sin \phi=1$, and equation (5)(ii) becomes

$$x+Dx^3=A.$$

Now A is proportional to $i=A'i$ say, so that

$$\frac{x}{i} + D \frac{x^3}{i} = A' (10)$$

where the x 's are the maximum values,

This can be checked by plotting $\frac{x}{i}$ against $\frac{x^3}{i}$, when the slope of the line which should be obtained will give D , and the intercept on the y axis will give A' .

The necessary figures from Fig. 10 are collected in the following table:—

i mA	x	$\frac{x}{i}$	$\frac{x^3}{i}$
0.37	0.185	0.495	0.017
0.60	0.295	0.490	0.043
0.91	0.435	0.477	0.091
1.36	0.625	0.459	0.180
1.51	0.670	0.445	0.200
1.70	0.745	0.440	0.244

and the third column is plotted against the fourth in Fig. 11.

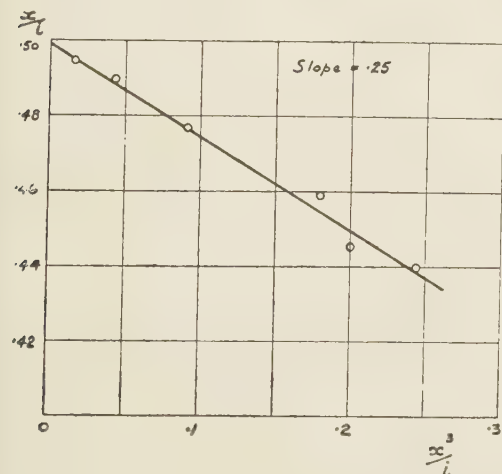


Fig 11.

It is seen that quite a good straight line results, that the intercept is $0.5=A'$, and that the slope is $0.25=D$, so that equation (ii) becomes

$$x + 0.25x^3 = 0.5i \quad . \quad . \quad . \quad (11)$$

Evidently therefore the theory successfully explains also the change of damping found with amplitude.

The decay curves of Fig. 7 also give a confirmation of the theory as regards the law governing the increase of resistance with amplitude, Δ plotted against x^2 giving in each case a straight line.

The remaining constants may now be found in the equations

$$\left. \begin{aligned} -m\omega^2x + sx - s'x^3 &= Ai \cos \varphi & \text{(i)} \\ \omega rx + \omega r'x^3 &= Ai \sin \varphi & \text{(ii)} \end{aligned} \right\} \cdot \cdot \cdot \cdot \cdot \cdot (12)$$

where A is now written for the force factor, and where in the previous work we have written

$$\frac{-m\omega^2 + s}{\omega r} = K\delta C, \quad \frac{s'}{\omega r} = B, \quad \frac{r'}{r} = D, \quad \frac{A}{\omega r} = A' \quad \dots \quad (12a)$$

Values have been found for D and B/K and A' .

Going back to equation (12) (i) above and dividing throughout by ωr gives

$$K\delta Cx - Bx^3 = A'i \cos \varphi \quad \dots \quad (13)$$

from which

$$\frac{x\delta\varphi}{\cos\varphi} - \frac{B}{K} \left(\frac{1}{\cos\varphi} \cdot x^3 \right) = \frac{A'i}{K} \quad \dots \quad (14)$$

If, therefore, $\frac{x\delta C}{\cos\varphi}$ is plotted against $\frac{x^3}{\cos\varphi}$, a straight line should result whose slope will be $\frac{B}{K}$ and intercept $\frac{A'i}{K}$. But first the values of $\cos\varphi$ must be found. This may be done from the second equation divided by ωr

$$x + \frac{r'}{r}x^3 = \frac{A}{\omega r} i \sin \varphi$$

which in the case of the curve of Fig. 16, with $i=1.51$ mA and using the values $\frac{r'}{r}=D=0.25$, and $\frac{A}{\omega r}=A'=0.5$, becomes

$$x + 0.25x^3 = 0.755 \sin \varphi.$$

From the figure constructed in this way it was found that the slope $= \frac{B}{K} = 1.4$

and the intercept $= \frac{A'i}{K} = 0.36$,

whence

$$K = \frac{A'i}{0.36} = \frac{0.5 \times 1.51}{0.36} = 2.1$$

and

$$B = 1.4 \times 2.1 = 2.94,$$

and the equations are

$$\left. \begin{aligned} 2.1\delta C - 2.94x^3 &= 0.755 \cos \varphi \\ x + 0.25x^3 &= 0.755 \sin \varphi \end{aligned} \right\} \dots \quad (15)$$

and

two equations for the determination of x and φ for given values of δC .

V. CALCULATION OF FORK CONSTANTS.

Taking the equivalent mass of one prong of the fork as $m=19$ grams (calculated) leads to an equivalent stiffness of

$$\begin{aligned} s &= m \times 320^2 \times 4\pi^2 \\ &= 77 \times 10^6 \text{ dynes/cm.} \end{aligned}$$

In the work which precedes the double amplitude was taken in mm. at a distance of 1.04 in. from the tip of the prong. To convert these to single amplitudes at the prong tips in cms. it is necessary to divide all values of x by 16.85 and correspondingly increase the constants in the equations.

Thus the equations (15) become

$$\begin{aligned} 2.1\delta C - 833x^3 &= 0.0448 \cos \varphi \\ x + 70.9x^3 &= 0.0448 \sin \varphi \end{aligned} \quad \cdot \cdot \cdot \cdot \cdot \cdot \quad (16)$$

Now $\delta\omega = 320 \times 2\pi \times \frac{\delta C}{2C_0}$ and the δC unit is $0.001 \mu F$, C_0 is

$0.510 \mu F$, or in the units chosen 510.

Hence $\delta\omega = 1.97\delta C$.

Also
$$\begin{aligned} \frac{-m\omega^2 + s}{\omega r} &= \frac{1}{r} \left(-m\omega + \frac{s}{\omega} \right) = \frac{m}{\omega r} (\omega_0^2 - \omega^2) \\ &= \frac{2m}{r} \delta\omega \quad \text{very nearly.} \end{aligned}$$

Hence
$$\frac{2m}{r} \delta\omega = \frac{2m}{r} \times 1.97\delta C = 2.1\delta C$$

whence
$$\frac{r}{2m} = 0.94 \text{ and } r = 0.94 \times 2 \times 19$$

$$= 35.7 \text{ dynes per cm. per second.}$$

Further, since $\frac{Ai}{\omega r} = 0.0448$, with $i = 1.51 \times \sqrt{2} \times 10^{-4}$ absolute units (the maximum value is used since the maximum value of x is used throughout)

$$\begin{aligned} A &= \frac{0.0448 \times 320 \times 2\pi \times 35.7}{1.51 \times \sqrt{2} \times 10^{-4}} \\ &= 15.1 \times 10^6 \text{ dynes/ab. amp.} \end{aligned}$$

Lastly,
$$833 = B = \frac{s'}{\omega r}$$

$\therefore s' = 833 \times 2\pi \times 320 \times 35.7 = 59.6 \times 10^6$

and
$$70.9 = \frac{r'}{r} \quad \therefore r' = 70.9 \times 35.7 = 2,530.$$

If the distorting effect is lumped together as $s_0 x^3 (\cos \xi - \gamma \sin \xi)$,

$$s_0 \cos \xi = 833\omega r$$

$$s\xi \sin \xi = 70.9\omega r$$

giving
$$\tan \xi = \frac{70.9}{833} = .085$$

and
$$\xi = 5^\circ,$$

which is an equivalent angle of lag, made up of electrical eddies and hysteresis and mechanical hysteresis.

The final equations for the motion x of the tip of the fork prongs are, therefore,

$$-19\omega^2 x + 77 \times 10^6 x - 59.6 \times 10^6 x^3 = 15.1 \times 10^6 i \cos \varphi$$

$$35.7\omega x + 2530\omega x^3 = 15.1 \times 10^6 i \sin \varphi$$

x being the amplitude in cms. and i the current amplitude in absolute units.

VI. DOUBLE FREQUENCY EXCITATION.

It appeared on consideration that if the current were of double the fork frequency, then terms in equation (2) with even powers of $\Phi_i x$ would now give a single frequency component of the pull. The most important of these would be $2\Phi_0\Phi_i \cdot 2bx$, which, introducing the time element, would be

$$4b\Phi_0\Phi_i \sin(2\omega t + \varphi) \sin \omega t = 4b\Phi_0\Phi_i x \left\{ \frac{1}{2} \cos(\omega t + \varphi) - \frac{1}{2} \cos(3\omega t - \varphi) \right\},$$

giving a single frequency component of amplitude $2b\Phi_0\Phi_i x$.

One would expect, therefore, that if the coil of the fork were supplied with a sufficiently powerful current of twice the frequency of the fork or round about that value, the fork once set into vibration would be maintained at a frequency exactly one half of that of the exciting current—i.e., at about its own natural frequency.

The experiment was successfully carried out on a 50 cycle double reed, excited by 20mA at about 100 cycles through an 8,000 turn coil, from an alternator in the first instance, but afterwards from a valve oscillator. There was no vibration until

the reeds were started by flicking with the finger, and then, within certain frequency limits, the amplitude built up until the reeds were striking the core.

The form of the resonance curve obtained appeared to be somewhat as shown in Fig. 12. To start the vibration the frequency had to be between f_1 and f_2 . On decreasing the frequency after starting the vibration, the amplitude increased until the reeds struck the core. At a certain frequency, f_3 , the amplitude suddenly dropped to zero. With frequencies between f_2 and f_3 the change from maximum amplitude (striking the core) to zero was always abrupt. If the amplitude was reduced by just touching the reed, the vibration collapsed. But quite stable amplitudes not striking the core existed between f_1 and f_2 , and this part of the curve could be traversed backwards or forwards by altering the frequency. If the frequency was lowered below f_2 , so that the reeds struck the core and then gradually increased, the reeds still struck, although f_2 had been passed. If now the reeds were very lightly damped by the finger the amplitude fell to a stable position along AB , but if the frequency had been raised beyond f_1 the amplitude fell to zero.

It was also observed that the greater the exciting current the greater the frequency range over which vibrations could be maintained.

Some progress towards an explanation of these results has been made.

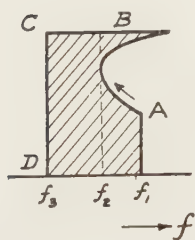


Fig. 12.

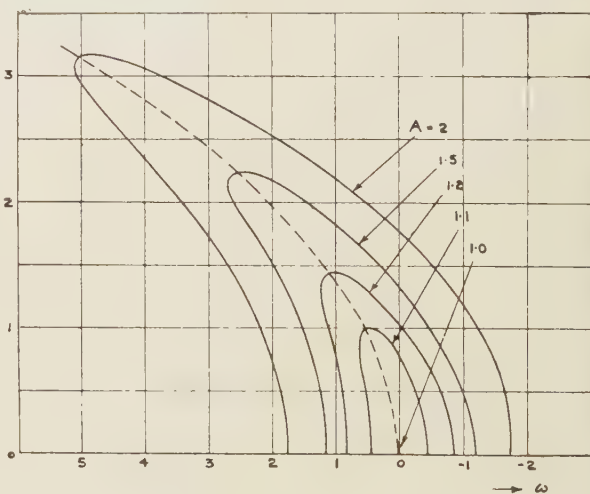


Fig. 13.

The differential equation is now

$$m\ddot{x} + r\dot{x} + sx = Aix.$$

Considering, as before, x as of standard phase (still assuming that its variations are sinusoidal), and writing the current as $i \sin (2\omega t + \varphi)$, and the displacement as $x \sin \omega t$, the equation becomes

$$\begin{aligned} -m\omega^2 x \sin \omega t + \omega r x \cos \omega t + e x \sin \omega t, \\ = Aix \sin (2\omega t + \varphi) \sin \omega t, \\ = \frac{Ai}{2} x (\cos \omega t \cos \varphi - \sin \omega t \sin \varphi), \end{aligned}$$

on neglecting all forces except those varying with angular frequency ω .

Equating sine and cosine terms gives

$$\left. \begin{aligned} -m\omega^2 + s &= -\frac{Ai}{2} \sin \varphi \\ \omega r &= \frac{Ai}{2} \cos \varphi \end{aligned} \right\} \dots \dots \dots (16)$$

Since x does not appear it can have any value whatever by this theory, provided that the second equation is satisfied by making $-Ai \geq 2\omega r$.

Then for any value of i

$$\left(\frac{Ai}{2}\right)^2 = (\omega r)^2 + (-m\omega^2 + s)^2, \dots \dots \dots (17)$$

which will fix the frequency, and

$$\tan \varphi = \frac{m\omega^2 - s}{\omega r} \dots \dots \dots (18)$$

which will fix the phase angle.

Thus a vibration with any given current is only possible at one particular frequency, and it will then have a definite phase angle with regard to the current. It will consequently be unstable.

The condition arrived at now is very similar to that in Melde's famous experiment and other cases of maintenance of vibrations with forces of double frequency, for which Rayleigh* gives the equation

$$\ddot{u} + k\dot{u} + (n^2 - 2a \sin pt)u = 0,$$

with solutions to a first approximation which can be identified with those given above.

A further step is the introduction into the equations of the x^3 terms found experimentally to be successful in the previous sections. The amplitude will now be fixed, and a vibration found to be possible over a range of frequencies and with varying phase angle.

* "Sound," I, p. 82.

Equations (16) become

$$\begin{aligned} -\omega^2 m x + s x - \frac{3}{4} s' x^3 &= -\frac{A i}{2} x \sin \varphi \\ \omega r x + D x^3 &= \frac{A i}{2} x \cos \varphi \end{aligned}$$

which may be written

$$\left. \begin{aligned} \frac{-m\omega^2 + s}{\omega r} - Bx^2 &= -A \sin \varphi \\ 1 + Dx^2 &= A \cos \varphi \end{aligned} \right\} \dots \dots \dots (19)$$

These equations have been examined arithmetically in the manner described in Section III, and the curves in Fig. 13 for values of $A=1.1, 1.2, 1.5$ and 2 are the result, taking $B=0.5$ and $D=0.1$.

These curves explain a number of the experimental results. It is clear, for

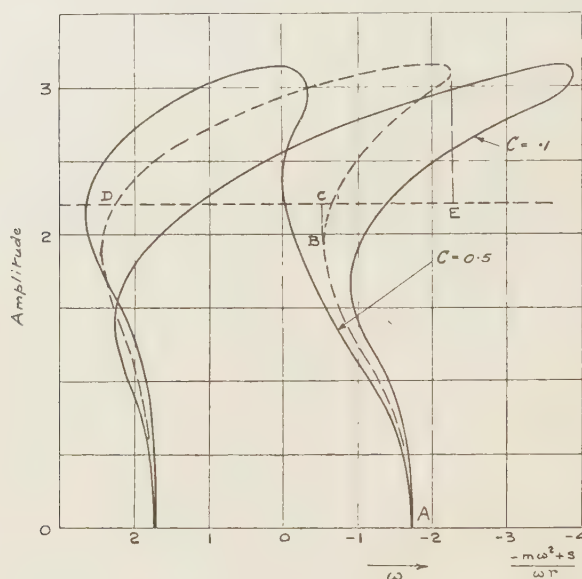


Fig. 14

instance, how the range of frequency over which vibrations are possible is greater the greater the current (and the value of A), and that as the frequency is lowered vibrations will increase in amplitude, and finally will cease abruptly (the undersides of the curves represent unstable conditions as before). But the sudden rise to large amplitude on decreasing the frequency and its persistence on again raising the frequency, as well as the necessity to give a starting flick, are unexplained.

A still closer explanation can be obtained by considering terms in the forces acting of higher powers of x . As the fork is vibrating at single frequency there will be a single frequency back E.M.F. produced proportional to x , which

will cause a current to flow through the coils. The term $\Phi_i^2 \cdot 2d \cdot x^3$ in the expansions of equation (2) will therefore contain a term proportional to $x^5 \sin^5 \omega t$, which will contain a fundamental term. As to its sign it will be more or less opposing the pull, and will be positive on the right-hand side of (19) (i). Its phase will be fixed relatively to x , and there will also be a damping term in equation (4) (ii), but this will be neglected as merely flattening the curve. Making this addition, therefore, equation (19) becomes

$$\left. \begin{aligned} \frac{-m\omega^2 + s}{r} - Bx^2 + Cx^4 &= -A \sin \varphi \\ 1 + Dx^2 &= A \cos \varphi \end{aligned} \right\} \dots \dots \dots (20)$$

with $A=2, B=0.5, D=0.1$ and $C=0.05$ and 0.1 , the curves in Fig. 14 are obtained.

These explain some of the outstanding points quite well if it is imagined that

at the dotted horizontal line the prongs strike the core. Referring to Fig. 14, lowering the frequency from A , the amplitude will rise steadily to B and then suddenly to C . On lowering further from C , the amplitude collapses suddenly at D . On raising the frequency from C a little and damping a stable position is found if the fall is on to the curve between A and B , but the collapse is complete if outside A . In any case the amplitude collapses at E .

Another effect of the double frequency excitation was found when the amplifier valve was overrun, so that a second harmonic was present in the fork-exciting current. On reversing the leads to the fork the amplitude of vibration changed quite considerably. This effect was noticed at all frequencies, but it was more marked near the resonant frequency. It also occurred with small amplitudes as well as with large ones.

This change of amplitude can be explained by a linear theory. In equation (2) the flux Φ_i due to the current can be written $\Phi_i \sin \omega t + \Phi_2 \sin (2\omega t + \psi_1)$, and in the expansion for the pull there will be terms $2\Phi_0 \Phi_i \sin \omega t$ and

$$2\Phi_0 \Phi_2 \cdot 2bx \sin (2\omega t + \psi_1) \sin \omega t,$$

yielding single frequency pulls, besides others in higher powers of x .

When the terminals are reversed we change the phase of the current, and therefore of the flux by 180 deg., so that the flux becomes

$$-\Phi_i \sin \omega t - \Phi_2 \sin (2\omega t + \psi_1).$$

But the sign of x must also be changed because the phase of the vibration will also be changed by 180 deg. Thus the pull becomes

$$-2\Phi_0 \Phi_i \sin \omega t + 4\Phi_0 \Phi_i bx \sin (2\omega t + \psi_1),$$

instead of

$$+2\Phi_0 \Phi_i \sin \omega t + 4\Phi_0 \Phi_i bx \sin (2\omega t + \psi_1).$$

The second term is acting in the reverse sense when the terminals are reversed.

To study the effect of such an impurity of wave form on the shape of the resonance curve, the equation of motion may be written

$$m\ddot{x} + r\dot{x} + sx = A_1 \sin (\omega t + \phi) + A_2 x \sin (2\omega t + \psi + \phi).$$

With $x = x \sin \omega t$ this becomes

$$-m\omega^2 x \sin \omega t + \omega r \cos \omega t + sx \sin \omega t$$

$$= A_1 \sin \omega t \cos \phi + A_1 \cos \omega t \sin \phi + \frac{A_2}{2} x \{ \cos \omega t \cos (\psi + \phi) - \sin \omega t \sin (\psi + \phi) \},$$

which gives

$$x^2 = \frac{A_1^2}{\sqrt{(-m\omega^2 + s)^2 + (\omega r)^2 - \frac{A_2^2}{4} \cos^2 \psi + \frac{A_2^2}{2} \sin \psi}},$$

and

$$\tan (\phi + \psi) = \frac{\omega r}{-m\omega^2 + s}.$$

From these expressions it appears that the amplitude depends upon ψ . If ψ is $+\frac{\pi}{2}$, the amplitude is decreased, if $-\frac{\pi}{2}$ it is increased, if 0 it is increased.

In the first and second cases $\tan \phi = \frac{\omega r}{-m\omega^2 + s}$ since $y=0$, but in the third and in all intermediate cases ϕ will be different, so that at resonance ϕ is not $\frac{\pi}{2}$. In all cases, however, both the amplitude and the phase changes will be symmetrical about the resonance value of ω , but as in no case is the resonance simple, the circle and straight line construction cannot succeed.

It may be noted that in the second and third cases, if $\frac{A_2}{2} = \omega r$, the amplitude at resonance is ∞ .

This corresponds to the conditions examined in equation (16).

When the amplitudes are large, so that the effects of the higher powers of x are to be taken into account, a still more complicated resonance curve must result.

VII. DISTORTIONS OF SECOND TYPE.

Everything that had been done up to the present indicated that with very small amplitudes the resonance curve of the fork would be practically symmetrical, and it was hoped by using a Kennelly mirror to measure the amplitudes and by measuring also the impedance of the fork to calculate the small amplitude fork constants in a similar manner to the methods used by Kennelly on the telephone receiver. But the resonance curve obtained in the manner shown in Fig. 15. The circle and straight line construction carried out on this curve gave two lines of unequal slopes for the $\tan \alpha/\omega$

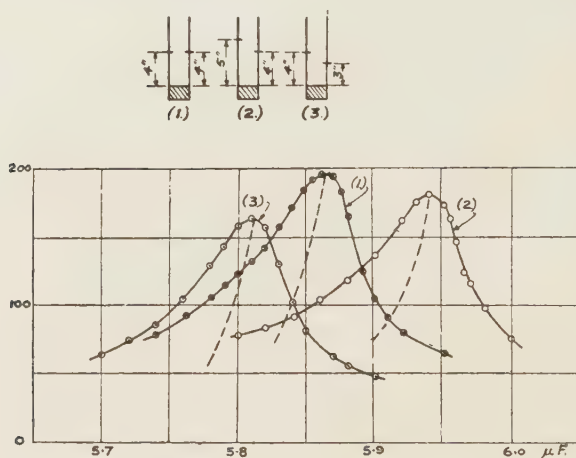


Fig. 15.

curve meeting near the resonance value, and the same form was given by the construction carried out on the impedance/frequency curve. With the Kennelly mirror removed, however, a perfectly straight line was obtained from the impedance/frequency curve. Evidently the distortion at the very small amplitude (about 40 microns) in the experiment was not inherent in the fork itself, but was produced by the Kennelly mirror.

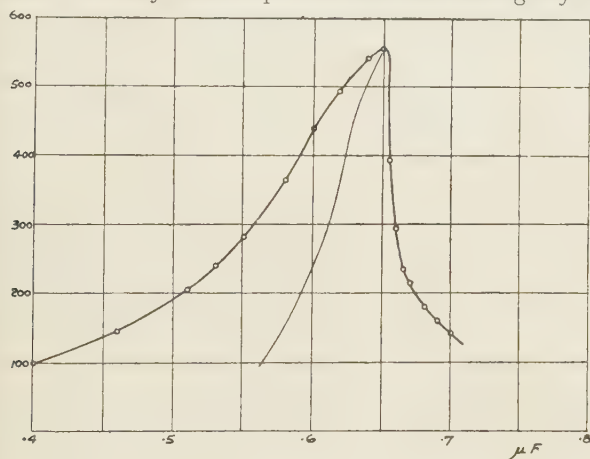
The same type of resonance curve has also been found with a laboratory-made fork, a vibration galvanometer, telephone receivers, and a single reed of a multiple reed frequency meter. It appears to be a "coupled circuit" effect, with the coupling either insufficient to give a double hump or involving too much addition to the damping. In some cases, however, the double hump does appear, as found by Kennelly for the telephone receiver and R. L. Jones* for the vibration galvanometer.

* Proc. Phys. Soc., Vol. 35, p. 67.

(i) *The Laboratory made fork* was simply two bars of hard steel clamped to a fairly massive iron block. The position of the block could be moved so as to alter the prong length, and the fork was supported on a wooden stand. With a fork length of $8\frac{1}{2}$ in. (frequency 120), the curves of Fig. 15 were obtained with riders in different positions along the prongs. It is seen that the main effects of throwing one prong or the other out of balance are to move the curve bodily and to increase the damping a little. The form of the curve as indicated by the slope of the mid-line remains substantially the same. Even when the out of balance was made very much greater this remained the case.

The amplitudes of the curves of Fig. 15 are comparatively small. In Fig. 16 the exciting current has been increased to make the amplitude three times as great, and the unstable effect has just appeared. The shape of the mid-line shows clearly the presence of both types of distortion.

The amplitude of the two prongs as read by the microscope was very nearly the same. A very thin strip of indiarubber was lightly stretched between the two prongs,



and attached by rubber solution. The microscope showed clearly that the centre of the rubber—that is, the point mid-way between the two prongs—was stationary, even when the fork was vibrating vigorously. The centre of mass of the system does not move. It does not follow that the actual amplitudes of vibration of the prongs with regard to the block to which they are rigidly clamped are identical. They may be different both as regards amplitude and phase,

the vibration of the block on its support to the stand being such as to keep the centre of mass stationary. The amplitudes that are read by the microscope are thus compounded of the motion of the prong and that of the whole fork, and these amplitudes are identical.

If in Fig. 17 (i) a_0, b_0, c_0 represent the positions of the prongs and the centre line at the tip of the prongs at rest from some fixed point, then $c_0 = \frac{1}{2}(a_0 + b_0)$. If the vibration of the left-hand prong is $a \sin(\omega t + \varphi_1)$ and of the right-hand prong $b \sin(\omega t + \varphi_2)$, and of the whole fork about the centre line $c \sin(\omega t + \varphi_3)$, then the position of the centre of the prongs at any moment is given by

$$\frac{a_0 + a \sin(\omega t + \varphi_1) + b_0 + b \sin(\omega t + \varphi_2)}{2} + c \sin(\omega t + \varphi_3),$$

and the movement by

$$\frac{1}{2}\{a \sin(\omega t + \varphi_1) + b \sin(\omega t + \varphi_2)\} + c \sin(\omega t + \varphi_3).$$

This movement as has been shown experimentally is zero, hence

$$a \sin(\omega t + \varphi_1) + b \sin(\omega t + \varphi_2) + 2c \sin(\omega t + \varphi_3) = 0. \quad \dots (21)$$

The amplitudes that are measured from a fixed reference point are the maximum values of

$$a \sin (\omega t + \varphi_1) + c \sin (\omega t + \varphi_3),$$

and

$$b \sin (\omega t + \varphi_2) + c \sin (\omega t + \varphi_3),$$

which (using (21)) are equal to

$$\pm \left\{ \frac{1}{2} a \sin (\omega t + \varphi_1) - \frac{1}{2} b \sin (\omega t + \varphi_2) \right\},$$

respectively.

This is shown vectorially in Fig. 17 (ii), where Oa and Ob represent the vibrations of the left and right hand prongs about the supporting block, with a vector sum Oc' . Oc drawn in the opposite direction to Oc' and half its length represents the motion of the whole fork about the stand. OA is the vector sum of Oa and Oc and OB that of Ob and Oc . OA and OB are the amplitudes that are measured, and they are equal. It is also clear that OA is the vector sum of $\frac{1}{2}Oa$ and $-\frac{1}{2}Ob$.

The construction thus found is used in Fig. 18 to find the form of the observed resonance curve when the prongs are out of balance both as regards natural frequency and damping. Rays drawn from O_1 and O_2 to AB represent the mechanical impedances of the two prongs and their intercepts on the circles represent the vector velocities (with phase angles multiplied by -1). Since the excitation of the two prongs is always 180 deg. out of phase the required observed amplitude is to be obtained by adding the circle intercepts instead of subtracting them. Thus, at the point ω' , O_1a_1 represents the motion of the first prong and O_2a_2 that of the second. a_2a_1' is drawn equal and parallel to O_1a_1 and O_2a_1' set out horizontally from ω' to give a point r' on the resonance curve.

It is seen that the shape of the curve is similar to those obtained experimentally, but that the slope of the mid-line is reversed as regards frequency. To obtain a slope in the right sense the prong with the higher natural frequency must have the higher damping, and it would appear that this condition must hold generally.

Explained on this basis the distortion of the curve of the Koenig fork by the Kennelly mirror would be due to an increase of the stiffness of the prong by the spring of the mirror, and at the same time an increase of the damping of the prong.

(ii) Resonance curves obtained from a vibration galvanometer of the bifilar type were of similar shape. As the current is increased the curve is moved bodily to a lower frequency. The conditions here are somewhat similar to those in the home-made fork, the observed amplitude depending on the motion of each of the stretching wires and upon any movement of the whole system. The lowering of frequency can be ascribed in a general way to an increase of coupling with current, but much further work remains to be done before the curves can be explained completely.

(iii) In the tuning fork and the vibration galvanometer there are clearly two separate members of the system which can have different resonant frequencies and produce by their interactions the "coupled circuit" effects. But in the cases of

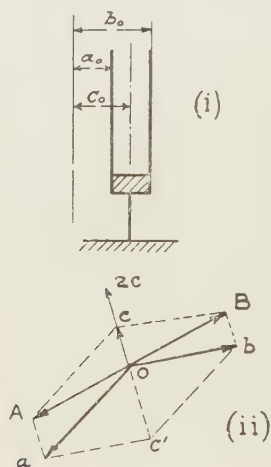


FIG. 17.

the telephone receiver diaphragm and a single reed, which can give similar curves, it must be imagined that the single member is capable of two slightly different modes of vibration in order to explain the results. Both the telephone receiver and the single reed show also distortions of the first type so that the unstable effect can be produced. This must have a great effect on the performance of loud-speakers when over-loaded.

CONCLUSION.

The resonance curve of apparently simple mechanical vibratory systems are in general distorted; if a vertical line is drawn through the maximum amplitude

in a curve of amplitude of vibration plotted against the frequency of excitation, then the curve is not symmetrical about this line. This occurs in such systems as tuning forks, reeds single or double, vibration galvanometers, and telephone receiver diaphragms.

It has been found that the lack of symmetry may be of two different types. The first case is that of a device acting as a perfectly simple or single system, which with very small exciting forces and consequently very small amplitudes, does give a symmetrical resonance curve. But with larger forces and consequently larger amplitudes distortion of the curve appears. In the Koenig tuning fork investigated, the distortion has been shown to be due to a decrease of resonant frequency and an increase of decay factor with amplitude. These changes are brought about by the

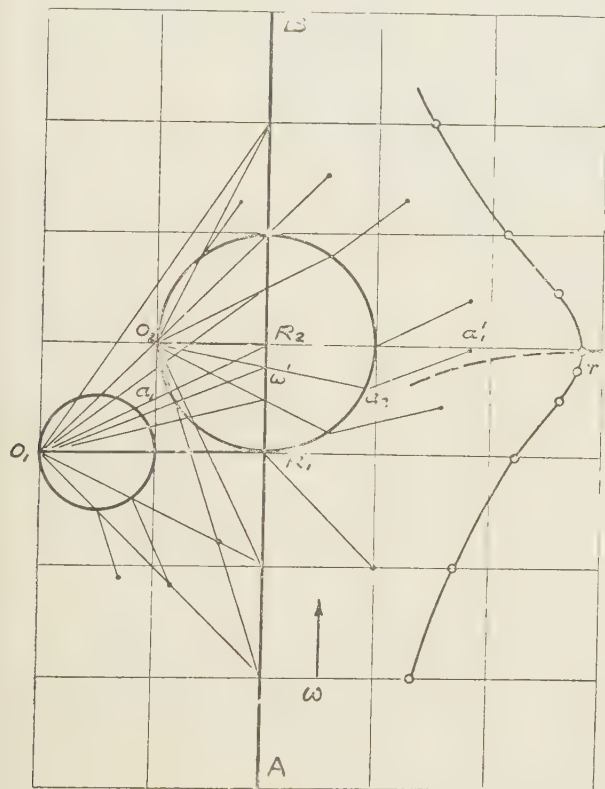


FIG. 18.

non-linearity with amplitude of the forces brought into play. The complete explanation is very complicated. Both the mechanical restoring forces and the electro-magnetic forces of the driving arrangement show this non-linearity, so that the distortion of the resonance curve is due in part to one and in part to the other. It is shown, however, that a close approximation to the form of the resonance curve found is obtained if in the equation of a simple driven system $m\ddot{x} + r\dot{x} + sx = f$, s is replaced by $(s_1 - s_2x^2)$, and r by $(r_1 + r_2x^2)$. The separation of

the mechanical and electro-magnetic components of the distorting forces has not been completed, and further work remains to be done here. It has, however, been found that all the factors s_1 s_2 r_1 and r_2 depend upon the nature of the support of the fork on the one hand, and on the position of the exciting coil, whether near the tips of the fork or nearer the base, on the other hand, so that the relative magnitudes of the two disturbances will be very different in different cases.

It has further been found as a natural consequence of the non-linearity, that unstable effects may be produced, so that the amplitude of vibration depends upon whether the frequency in question has been approached from a higher or a lower frequency, and that excitation of large amplitude vibrations of fundamental frequency can be carried out by currents of double frequency.

The second cause of dissymmetry of resonance curves is a "coupled circuit" effect, and occurs at small amplitudes as well as large ones. The mechanical system even though apparently single or simple, has certain inherent out of balance which makes it in effect two individual members coupled together. The form of the resonance curve in this case is different from that of the first; the mid-line of the curve bends over at the top in the first case and is vertical at the bottom. In the second case it is vertical at the top and bends over at the bottom.

Even though the system shows this second type of distortion, the first type appears when the amplitudes are large. The mathematical form of the resonance curve must then be very complicated.

It appears, therefore, that for use as a frequency meter, or for fixing the wavelength of a wireless station from a valve-maintained tuning fork, or for any purpose in which it is important that the frequency of the fork should be as nearly absolutely constant as possible, it is necessary to work with very small amplitudes of vibration.

The whole of the experimental work was carried out in the Telegraphy and Telephony Laboratories of the City and Guilds (Eng.) College, where valuable assistance was given by Mr. I. L. Turnbull, a senior student, in many of the experiments, and by Mr. F. W. Andrews in the design and construction of the apparatus. The author was throughout indebted to Prof. C. L. Fortescue for ever-ready discussion and suggestion.

DISCUSSION.

Mr. ALBERT CAMPBELL: It is interesting to know that tuning forks when driven at large amplitudes may show the condition of instability described by the author. I have observed similar instability with strip vibrators of thin steel, used in a frequency meter of adjustable reed type (Phys. Soc. Proc., 14, p. 267, 1896). The resonance point is slightly different with ascending and descending pitch. I would also draw attention to the interesting series of resonance curves given in Dr. R. Hartmann-Kempf's book, "Elektro-Akustische Untersuchungen" (Gebr. Knauer, Frankfurt A.M., 1903). Dr. Mallett is to be congratulated.

XXVIII.—ATOMIC NUCLEI AND THEIR TRANSFORMATIONS.

The Twelfth Guthrie Lecture. Delivered February 25, 1927.

By Sir ERNEST RUTHERFORD, O.M., P.R.S.

IN the year 1911, when I put forward the nuclear theory of atomic structure based on the interpretation of the simple scattering of α particles by matter, there seemed little hope of any immediate progress in our knowledge of the detailed constitution of the nucleus. So much was this the case that in a discussion on atomic structure before the Royal Society in 1913, in answer to an enquiry as to the constitution of the nucleus, I replied that a consideration of this question had best be left to the next generation. While much more progress has been made in the interval than seemed likely at that time, we cannot but recognise that only a beginning has yet been made in the attack on this most difficult and fundamental of problems in Physics. It may be of interest, however, to give a brief résumé of the main additions to our knowledge in recent years, and then later to consider in more detail some of the more promising lines of attack.

From the earlier experiments on scattering of α particles, it seemed certain that the space occupied by the nucleus of an atom was minute compared with that occupied by its whole structure. Consequently, the constituent parts of the charged nucleus must be held together in this minute volume by such powerful forces that we could not hope to influence them seriously by the physical forces available in the laboratory. The great magnitude of the forces in the nucleus of a heavy atom was clearly manifested by the enormous individual kinetic energy of the α and β particles set free during the radioactive transformations. Later, by the work of Ellis, Meitner and others, definite evidence was obtained that the greater part of the γ radiation also had its origin in the nucleus. The wave-length of some of these γ rays was measured, thus giving us definite information of some of the modes of vibration of the nuclear constituents. The individual quantum energy of some of these γ rays is very high, corresponding in some cases to at least 3 million electron volts.

The wonderful series of transformations manifested in the elements uranium, thorium and actinium provided us with a wealth of data on the modes of transformation of these heavy elements; but, unfortunately, our theories of nuclear structure are as yet in such an embryonic state that we are able to make little if any use of the numerous facts that have been accumulated. I shall refer later to this subject, but it suffices at present to point out that the study of these transformations did give us some general information of great importance about nuclei.

Since both helium nuclei (α particles) and swift electrons (β particles) are hurled from the nucleus, it could safely be deduced that the nucleus of these heavy atoms must contain electrons and helium nuclei as part of their structure, unless it be supposed that the helium nucleus is in some way formed of simpler constituents at the moment of its expulsion from the main nucleus.

In another direction, the study of the radioactive transformations gave us the most convincing evidence in support of the nuclear theory, and brought to light

the extraordinarily simple relations which exist between the change in atomic number in a disintegrating element and the nature of the transformation. The expulsion of an α particle, which carries a charge $2e$, lowers the nuclear charge by two units, while the expulsion of an electron raises the nuclear charge one unit. This generalisation, known as the Displacement Law, first put forward in general form by Russell, Fajans and Soddy, gives us at once the nuclear charge and mass of each element in the radioactive series, and thus fixes the ordinary physical and chemical properties of each radioactive element, as well as its atomic weight. This relation is remarkable both for its simplicity and generality, and to my mind represents a contribution to our knowledge of nuclei of the greatest importance and interest.

I can only refer in passing to the great step in advance made by Moseley in indicating that the nuclear charge of an element in fundamental units is represented by its ordinal or atomic number. This deduction has been directly verified by Chadwick in several cases by accurate measurement of the scattering of α particles.

The discovery of elements in the radioactive series, now known as isotopes, which are chemically identical but differ in mass and radioactive properties, was a notable addition to our knowledge. It was at once seen that the identity of chemical behaviour was a definite proof that isotopes had the same nuclear charge, while the difference in mass and radioactivity showed that the nuclei of the isotopes differed in constitution and stability. The difference in mass of some of the radioactive isotopes is noteworthy. For example, in these radioactive series there are believed to be 7 isotopes of lead, which vary in mass between 214 and 206, while their half-periods of transformation for the radioactive isotopes vary between 27 minutes and 16 years. The end products, uranium-lead (206) and thorium-lead (208), are believed to be permanently stable.

I need only refer in passing to the proof by Aston that many of the ordinary elements consist of a mixture of isotopes. One of the important deductions from his work is the "whole-number" rule, for the mass of each isotope is found in many cases to be nearly a whole number expressed in terms of the mass of oxygen taken as 16. This indicates that the ultimate building unit of nuclei is of mass unity in the nuclear structure. This unit, known as the proton, is slightly less in mass than the hydrogen nucleus mass 1.0072, the difference being ascribed to the packing effect—that is, to the interaction of the electromagnetic fields in the highly condensed nucleus. Later experiments have shown, however, that while the whole-number rule holds very approximately in many cases, it is widely departed from in others. The accuracy of the "whole-number rule" is now under examination by Dr. Aston, using a mass spectrograph of much greater resolution, so that we may soon expect to have much more precise information as to the relative masses of the isotopes. If these constants can be obtained with sufficient precision, they must certainly prove of great value in throwing light on many points of nuclear structure. The reason of this will be clear from the discussion later in this Paper.

We have seen that radioactive evidence indicates that helium nuclei and electrons are constituents of the nuclear structure of heavy atoms; but, notwithstanding the fundamental character of the radioactive transformations, in no case is a proton liberated from the nucleus. The general evidence strongly supports the view that the electrons and protons are the fundamental units of nuclear structures, and this

has been directly confirmed by the experiments of Rutherford and Chadwick, Petersen and Kirsch, on the artificial disintegration of light elements by α particles. For twelve elements definite proof has been obtained that protons are ejected at high speed when the material is bombarded by swift α particles. With the exception of carbon and oxygen, on which the experimental evidence is conflicting, all elements from boron to potassium inclusive liberate protons under bombardment.

In the majority of these experiments, the scintillation method has been employed where certain observations can only be made of the emission of particles which have a range greater than the scattered α particles, for the number of the latter are usually greatly in excess of the number of protons ejected. In general, the protons appear to be emitted about equally in all directions relative to the bombarding pencil of α particles; but the speed of the protons is greater in the forward than in the backward direction, probably due to the motion of the bombarded nucleus.

Unfortunately, the amount of disintegration effected by even the swiftest α particles is very small, only one proton being liberated for about 100,000 bombarding α particles. The experimental study of this artificial disintegration of elements, while simple in principle, is in practice beset with many experimental and observational difficulties, while an elaborate technique is required to obtain quantitative results.

DIMENSIONS AND LAW OF FORCE.

We have seen that the study of the scattering of α particles first disclosed the nuclear structure of atoms, and even now this is the only trustworthy method of investigation of the law of force in the neighbourhood of a nucleus, and for giving some idea of the dimensions of the nuclear structure. In the classical experiments of Geiger and Marsden, the scattering of α particles at various angles was found to be in accord with the coulomb law of force for the elements silver and gold. This question has recently been re-examined by Rutherford and Chadwick by a modified method with similar results. Even for the swiftest α particles available, the scattering of α particles through an angle of 135° for the elements copper, silver and gold showed no observable departure from that calculated on the inverse square law. For these elements the closest distance of approach of the α particle to the nucleus using α particles from radium C of velocity 1.922×10^9 cms./sec. is 1.23, 1.94, 3.15×10^{-12} cms. for copper, silver and gold respectively. Since it is to be expected that the coulomb law of force would be departed from if the α particle entered the main nuclear structure, we can thus conclude that the radius of the charged nucleus, assumed spherical, must be less than the above numbers for copper, silver and gold respectively. Until α particles of higher speed are available there is no way of fixing the minimum limit of the nuclear dimensions by these methods. Similar experiments made with a thin sputtered layer of uranium metal indicated no certain change of the law of force at a distance 3×10^{-12} cms. from the centre of the nucleus. This observation raises a considerable difficulty, for it will be seen later that certain radioactive data indicate that the uranium nucleus, or at any rate some constituents of its structure, must extend to more than twice this distance.

In the case of the light atoms of small nuclear charge, the α particle in a close collision can approach much nearer to the nucleus, and we are thus enabled to study the law of force at much smaller distances. The experiments of Rutherford and the detailed investigations of Chadwick and Bieler early disclosed that the

coulomb law of force was no longer applicable when swift α particles collided with hydrogen nuclei. The variation of scattering with angle as well as the variation with velocity of the α particle, are almost the reverse of that to be expected on an inverse square law. For the swiftest α particles, the number of H particles shot forward within a few degrees of the direction of the α particle is about a hundred times the calculated number.

The nature of the results obtained may be illustrated by the following example. The number of H particles projected after collision in directions making angles from 20° to 30° with the line of flight of the incident α particles was counted. When the velocity of the α particle was small, range about 2 cms. in air, the number of α particles scattered in this specified direction was about that to be expected on a coulomb law of force. With increase of velocity of the α particle, the number of H particles observed was greater than the theoretical, and the discrepancy increased very rapidly with rise of velocity of the incident α particles. For example, with α particles of 2.9 cms. range, the observed number was three times the calculated, while for α particles of 6.6 cms. range, the ratio rose to nearly thirty.

This failure of the inverse square law was interpreted by Chadwick and Bieler in the following way: Assuming that the H nucleus could be regarded as a point charge, a model was sought for the He nucleus (α particle), which would account in a general way for all the experimental results. It was found, from the calculation of Darwin, that an α particle behaved in these conditions as a body with properties intermediate between those of a charged elastic sphere and a charged electric plate, and it was compared as a first approximation, with an elastic oblate spheroid of semi-axes about 8×10^{-13} cm. and 4×10^{-13} cm., moving in the direction of its minor axis (see also p. 369). An H nucleus projected towards such an α particle would move under the ordinary electrostatic forces until it reached the spheroidal surface of the above dimensions. It would thus encounter a very powerful field of force, and recoil as from a hard elastic body. It was not possible to make any close comparison with this model, owing to the difficulty of calculating the collision relations for such a spheroid.

Looking at this model from a physical standpoint, we may say that the scattering is normal for all particles which do not enter the spheroidal surface, but any particle which crosses this surface encounters a deflecting field in which the forces on the particles increase much more rapidly than is to be expected on an inverse square law of force, so that the scattering is abnormal.

Before discussing this model in more detail, we shall now consider some experiments which Dr. Chadwick and I have recently made to examine the scattering of α particles in helium by a similar method. In this case, the particles concerned in the collision are identical in charge and mass, and have the same structure. There is, however, one difficulty in these experiments which does not arise in the investigations of the collision between α particles and H nuclei, where the recoiling H particles on account of their greater range can in most cases be observed independently of the incident α particles. In the case of a collision between two particles of equal mass, the angle between the scattered α particle and the recoiling nucleus is always 90° , and there is no way of distinguishing between the two types of particles. For example, suppose we are observing the particles which are scattered at an angle ϕ with the direction of the incident particles. These will include the α particles scattered through an angle ϕ and the recoiling nuclei which arise from a

particles scattered through $90^\circ - \phi$, and in many cases, where the law of inverse square is no longer applicable, we can only estimate roughly the relative importance of the two groups.

Taken as a whole, the distribution of scattered particles in collisions between α particles and He nuclei is very similar in type to that observed between α particles and H nuclei. There is a similar concentration of scattered α particles in the forward direction, which is the more marked the swifter the incident α particle.

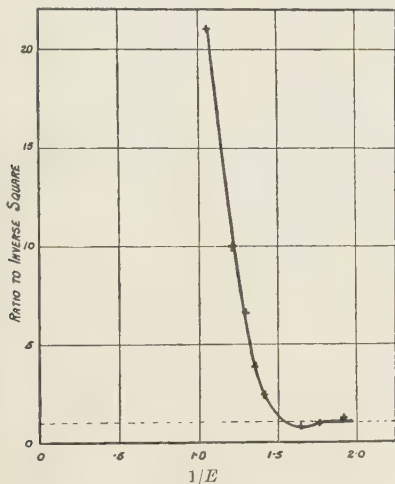


FIG. 1.

In one respect, these experiments gave us information which could not easily be obtained from the investigation of H nuclei. When the scattered particles are observed between 40° and 50° with the incident rays, the number of scattered α particles and the helium nuclei set in motion by recoil are equal in number and velocity. The curve of variation of number with the energy of the incident α particles is indicated in Fig. 1, where the ordinates represent the ratio of the observed to the calculated number and the abscissæ the value of $1/E$, where E is the energy of the colliding α particles. The dotted line for ordinate 1 represents the relation that would be observed for an inverse square law. It is seen that the ratio is large for swift α particles, falls rapidly below unity and then rises again to the calculated value.

A much more marked drop below the calculated value is observed for α particles of range about 3 cms. when the scattering is observed between 10° and 20° .

It is thus clear that for a definite range of velocity of the α particle, the scattering between certain angles is greater than the "normal" value—i.e., the value calculated for point charges—i.e., obeying the inverse square law. For another range of velocity, the scattering falls below the normal, rising ultimately to the normal value as the speed of the α particles decreases.

This deficiency of scattering between certain angles for α particles of given velocity is to be expected for any type of collision, where the scattering is abnormally large for another range of angles of scattering. As this conclusion is of a very general character and quite independent of any knowledge of the forces involved in a collision, it may be desirable to discuss it in some detail.

In Fig. 2 is represented a central section of an imaginary surface which represents the boundary dividing the region where the forces due to a nucleus are normal and abnormal. Any particle which does not enter this surface suffers normal scattering, but any particle swift enough to penetrate within the surface is deflected from its path by some unknown combination of forces. Suppose, for simplicity, the α particles are projected normally to the section in Fig. 2. This section con-

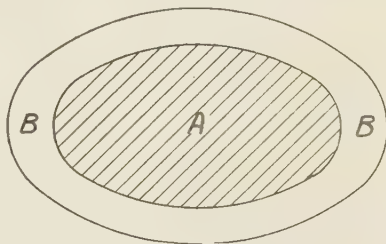


FIG. 2.

quently represents the target area exposed to the α particles. Suppose, for illustration, that all particles which fall within the inner area A are scattered within a certain range of angles, and that the number of α particles so scattered is above normal. Since the number of α particles falling on the surface is fixed by its target area, and each particle is scattered in one direction or another, the excess of scattered particles due to the target area A must be exactly counterbalanced by a defect in the number scattered through another range of angles by the region B . This must always be true for a *given velocity* of incident α particles provided that no α particles are absorbed by the nucleus.

In some respects, this conclusion that an excess of α particles in one direction must be balanced by a defect in another direction is an obvious deduction, but the utility of this conception in interpreting the phenomena of scattering has not, so far as I am aware, been fully recognised. It can only be applied when the angular distribution of scattered α particles has been determined for a definite velocity of incident α particles, and has the great advantage that it does not involve any knowledge of the nature of the collision or of the structure of the nucleus.

For example, Chadwick and Bieler examined the scattering of H particles for

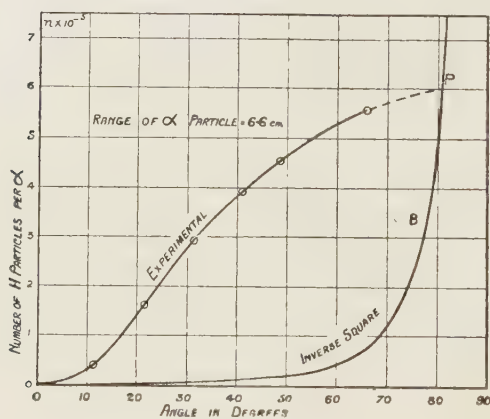


FIG. 3.

α particles of range 6.3 cms. in air between the angles 0° and 60° . The curve obtained by them is shown in Fig. 3, where the ordinates represent the ratio of the number of H particles to incident α particles. Ultimately above a certain angle of deflexion the scattering must become normal. The value calculated on an inverse square law is shown in curve B . Extrapolating the observed curve, the two intersect at P for an angle of about 80° , which represents the division between the two angular regions where the scattering is normal and abnormal. Since there is a large excess of α particles scattered within

the angles of 0° to 60° , there must be a corresponding defect in number between 60° and 80° . This is illustrated in Fig. 4, where the number scattered per degree from 0° to 80° is compared with the theoretical number. The area AA above the theoretical curve must be balanced by the area between this curve and BB . Assuming the correctness of the data involved, it is clear that there must be a marked defect in the scattering about 70° , and that the curve rises rapidly about 80° , where the scattering becomes normal.

On account of the shortness of the range of the H particles scattered at such large angles, it is difficult to verify this deduction experimentally, but there is little doubt of its essential correctness.

In this connection it is of especial interest to consider the scattering of α particles by aluminium, a subject which has been carefully studied the last few years. Bieler compared the scattering by thin sheets of aluminium and gold, and assuming that the scattering of gold was normal, concluded that the number of α particles scattered

by aluminium was less than the normal. This defect from the calculated value increased with the speed of the α particles and the angle of scattering of the α particles. Rutherford and Chadwick examined the scattering at larger angles—viz.,

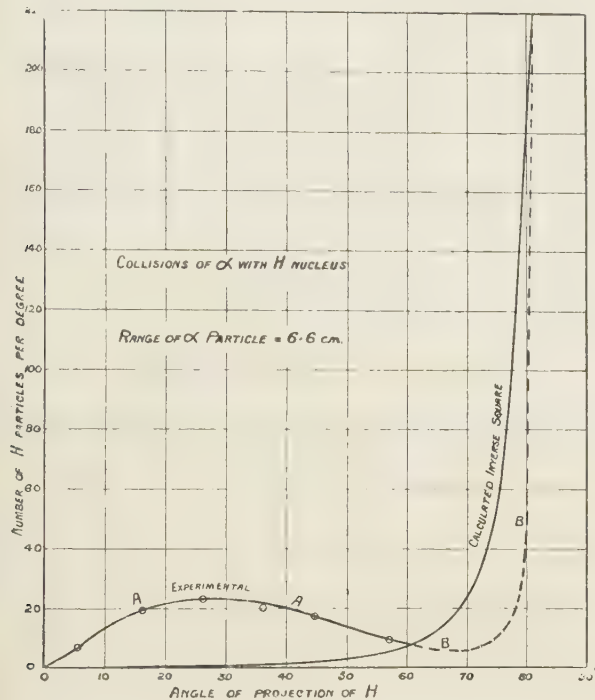


FIG. 4.

showed a marked defect from the normal, but no minimum was observed. There can be little doubt that if still higher range α particles could have been used, the curve would have passed through a minimum corresponding to Fig. 5.

Using the data published by Bieler, Rutherford and Chadwick, it is possible to estimate the defect of scattering for a 5 cm. α particle for all angles between 30° and 135° . The results are plotted in Fig. 6. It is seen that there is a marked defect of scattering over this wide range of angles. From what has already been said, it is clear there must be an excess of scattering in another angular region to compensate it. It does not seem likely that the scattering round 180° can be sufficiently

large for such a compensation, for it would mean an increase in this region of the order of one hundred times the calculated value. It is not easy to investigate

135° —and found an unexpected result, which is illustrated in Fig. 5, where the dotted line represents the normal calculated scattering and the full curve the experimental results. The abscissæ represent the reciprocal of the energy of the α particle, so that the left side of the curve represents high velocities of the α particles.

For low energies, the ratio is less than unity and decreases with increase of energy of the α particle, passing through a minimum corresponding to α particles of about 5 cms. range. For the highest range of particles used—viz., 6.6 cms.—the scattering curve is rising steeply. Similar results were observed for magnesium. For 90° , the scattering curves

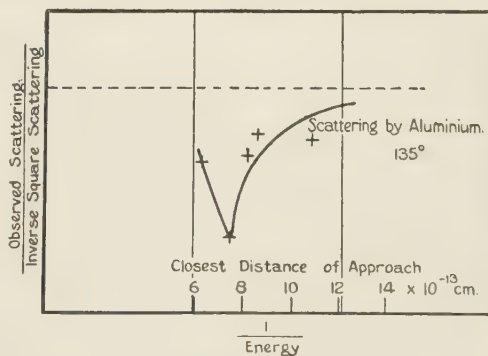


FIG. 5.

this point, but the experiments now in progress ought to settle whether any marked excess is observed. Failing compensation in this region, there must be an excess of scattering in the region between 0° and 30° . This is under investigation, but it may be difficult to obtain results of an accuracy required to settle this point, since a small percentage excess of scattering at a small angle may easily compensate for the defect over a range of 100° shown in the figure.

The examples referred to all illustrate in an interesting way the usefulness of this principle of compensation and the suggestions it offers of useful directions of investigation.

The principle is only valid if there is no disappearance of α particles in the nucleus. Now it is known that protons are ejected from aluminium and magnesium

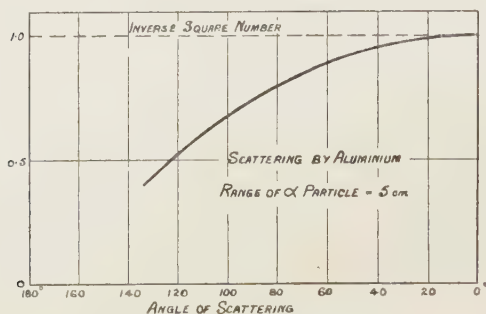


FIG. 6.

are too uncertain to give a decisive answer as to whether the defect of scattered α particles shown in Fig. 6 can be accounted for by capture of α particles. The evidence available rather indicates that the number of ejected protons is only a fraction of the defect in scattered α particles. If this be the case, we must probably look for the defect of scattering by an excess at other angles.

NUCLEAR FORCES AND STRUCTURE.

We have so far compared the observed scattering of α particles by nuclei with that calculated on classical mechanics. Assuming that the nuclei behave as point charges, any divergence from calculation has been assigned to departure of the forces from a coulomb law or to a nuclear structure comparable in dimensions with the distances involved in close collisions.

During the last year, however, the adequacy of classical mechanics for a solution of atomic problems has been questioned, and a new mechanics has arisen largely based on the work of Heisenberg, Schrödinger, Born, Dirac and others. The new mechanics has already shown its power by giving a correct solution of a variety of atomic problems, and it has been suggested that the abnormal scattering of α particles by light elements may be only apparent and due rather to a failure of the classical mechanics when applied to collisions than to any failure in the coulomb law of force.

Recently, however, Wentzel, Born and Oppenheimer have shown that on certain assumptions the laws of scattering by a central field of force obeying the law of inverse squares are of the same form on wave mechanics as on classical mechanics. A general attack on this problem of collisions is in progress, but I understand that

apart from minor perturbations it seems unlikely that the wave mechanics can offer any explanation of the abnormal scattering of high velocity α particles by the light elements hydrogen, helium and aluminium.

Schrödinger and De Broglie have given certain criteria for the conditions under which marked diffraction patterns in collisions may be expected. In a Paper in course of publication,* kindly shown me by Mr. Blackett, these criteria have been interpreted in terms of collision-relations founded on classical mechanics. Unfortunately the criteria are not very precise, and until some scattering phenomena have been completely explained on the wave mechanics, so that the requisite comparative data are available, it is difficult to be certain where marked diffraction patterns may be expected to disclose themselves in the type of collision we have been investigating, where swift massive particles collide with heavy nuclei. In general, if we try to fix the conditions by assuming, say, that the abnormal scattering of α particles by aluminium is explicable on wave mechanics, it is found that the criteria break down completely for collisions of α particles with hydrogen and helium nuclei.

It thus appears probable that we have to seek for an explanation of abnormal scattering of α particles in the detailed structure of the nucleus and the forces arising from its constituent particles. We have also to take into consideration the enormous distorting forces in the nuclei which must come into play in these very intense collisions where the nuclei approach very closely to each other. In my original Paper† on the scattering of α particles by hydrogen nuclei, I drew attention to the importance of these distorting forces in altering the normal structure of nuclei, and thus the forces in their neighbourhood, and pointed out that at short distances the forces might change from those of repulsion to those of attraction.

In order to account for the defect of scattering in aluminium, Bieler worked out mathematically the alteration of the scattering to be expected if the α particle were acted on by a combination of forces—viz., the ordinary repulsive electrostatic forces between the nuclei, and an attractive force varying as the inverse fourth power. In this way, he was able to account approximately for his experimental results with aluminium. This question has been recently investigated in more detail by Debye and Hardmeier on a slightly different basis. They assume that as the α particle approaches closely to the nucleus there will be a polarisation of the nucleus in the same way as a charged particle in approaching a neutral atom is able to polarise its structure. This polarisation involves an attractive force on the α particle, varying as r^{-5} , where r is the distance from the centre of the nucleus. The forces are supposed to be proportional to the volume occupied by the nucleus. By reasonable assumptions as to the constants involved, they have been able to calculate the scattering of α particles of different velocity when the scattering angle is 135° . In this way, they obtain a theoretical scattering curve for aluminium, which resembles in general features the curve shown in Fig. 5. The defect in scattering is similar to that observed, and there is the same pronounced minimum in the scattering curve. It thus seems not unlikely that the main peculiarities of the scattering of α particles by aluminium and magnesium may be due to polarisation

* Since published *Camb. Phil. Soc.*, 33, pt. 6, (1927).

† *Phil. Mag.*, 37, p. 561 (1919.)

of the nucleus by the charged α particle. It is, however, not likely that the attractive forces which arise in this way can be expressed by a simple power law, for the distances involved are of the same order as the dimensions of the nucleus which may not have a symmetrical structure. It would, however, be of great interest to work out the simple theory in detail to fix the scattering of α particles of given velocity for all angles. Such a calculation would give valuable evidence as to the angular region in which the number of scattered α particles is in excess of the normal in order to account for the defect at large angles already discovered.

Reference should be made to another point of interest in this connection. Since in a close collision of the types considered there should be considerable movement of the nuclear constituents which may not be moving in quantized orbits, energy may be dissipated in some form as a result of the collision. If this were the case, the ordinary laws of momentum and energy would not be applicable to such collisions. So far no conclusive evidence of such an effect has been observed, except in cases where the nucleus suffers disintegration. The velocity of the scattered α particle, as far as our experiments have gone, corresponds to that calculated on the basis of an elastic collision. The most definite method of testing this point is a careful examination of the photographs of forked tracks due to collisions obtained in an expansion chamber. Blackett has examined a number of such photographs, and within the small experimental error finds no evidence of a loss of energy even in very close collisions of α particles with light elements. This question is an important one, and even more accurate data are required to give a definite answer. It appears, however, fairly certain that the loss of energy due to such impacts cannot be more than a per cent. or two of the total energy of the α particle. It thus appears probable that the greater part, if not all, of the energy which is lost by the α particle in distorting the nucleus in the first stage of the collision is returned to the α particle as it leaves the nucleus. It should, of course, be borne in mind that the colliding α particle, as well as the recoiling nucleus, both suffer some distortion, and in the case of collisions of α particles with light atoms the distortion may be of the same order of magnitude for both nuclei.

We have already discussed some of the striking peculiarities in the scattering of α particles by the hydrogen and helium nuclei, and the mode of interpretation of the main features of the phenomena. There are, however, a number of other interesting points to which reference should be made. In the interpretation of the collisions between α particles and H nuclei it was supposed that the latter might be regarded as a point charge, and the "shape" of the α particle, or rather of the region where the forces are abnormal, was estimated. It is of interest to compare the results obtained with hydrogen and helium. As we have seen for very close collisions, the scattering is very abnormal in both cases, but becomes normal for sufficiently slow α particles where the closest distance of approach is of the order 4×10^{-13} cm. The general evidence indicates that, if anything, the scattering remains normal for an even closer distance of approach in the case of helium nuclei than with H nuclei. If the complexity of the forces were ascribed entirely to the α particle, while the H nucleus acted as a point charge, we should expect the scattering to become abnormal at about twice the distance for collisions of α particles with helium nuclei than for collisions with hydrogen nuclei. If anything, however, the shortest distance of approach appears to be even smaller for helium than for hydrogen. It is, of course, very difficult to estimate the effect of distortion, but

it would appear that the hydrogen nucleus or proton must have dimensions comparable with those of the α particle. This is an unexpected result, for it is usually supposed that the proton is of dimensions much smaller than the electron. Since, however, we are referring to the dimensions of the region where the forces are abnormal, rather than that occupied by the actual structure of the nucleus, it would be more accurate to say that the region round the proton where the forces are abnormal is comparable in dimensions with the corresponding region for the α particle.

It is, of course, very difficult to offer any definite explanation of the origin of the forces which are brought to light by a study of collisions, for it must be remembered that the nuclei are both in motion during the collision, and both are in an abnormal state due to the distorting forces arising from their close approach to each other. It may be, however, that the abnormal scattering is due, not only to the modification of the electrostatic forces by distortion, but that magnetic forces become very important in such close collisions, and, indeed, may play a predominant part in determining the scattering for light elements. During the last few years it has been suggested that the negative electron has a definite magnetic moment associated with it, and, if this be the case, it is not unreasonable to suppose that the proton has a corresponding magnetic moment. Frenkel, who has discussed the effect of magnetic forces on the structure of the α particle, supposes that the proton has a magnetic moment only $1/1840$ of the electron. Whatever view we may take on this question, it is clear that we may expect strong magnetic forces near the nucleus whether due to the intrinsic magnetic moment of its constituents or to their motion. While no doubt the particles constituting the nucleus may tend to adjust themselves to reduce the resulting magnetic moment to a minimum, there must always be strong local magnetic forces in the neighbourhood of the constituent parts of any nucleus.

In the case of the helium nucleus, which may be supposed to be made up of four protons and two electrons, it is not difficult to calculate from available data that the effect of these magnetic forces becomes relatively prominent for distances of the order of 4×10^{-13} cm.—a distance at which we know the scattering of α particles of hydrogen and helium nuclei become markedly abnormal. It is important to note, also, that since the deflecting force on a charged particle due to a magnetic field is proportional to the speed of the particle, we are able to understand in a general way why the abnormality in the scattering in hydrogen and helium increases so markedly with rise in velocity of the α particle.

It seems probable that the magnetic forces to be expected in the neighbourhood of a helium nucleus are of the right order of magnitude to account in a general way for the abnormal scattering observed in close collisions with an α particle. In order to explain the scattering of α particles by hydrogen nuclei it has been deduced that the region in which the forces due to the α particle are abnormal is spheroidal in shape, and that this spheroid must move with its short axis in the direction of the α particle. This indicates a definite orientation of the α particle which may be brought about during the collision by the turning couples due to the interaction of the magnetic fields due to the two nuclei. While, of course, such considerations are largely speculative in character, the general evidence does support the view that the region surrounding the nucleus must be the seat of intense magnetic forces which largely control the abnormal scattering which is observed.

RADIOACTIVE NUCLEI.

We shall first discuss a marked discrepancy between the dimensions of the nucleus of uranium when estimated from scattering and radioactive data. Rutherford and Chadwick found that the scattering of a film of uranium appeared to be normal—i.e., to correspond to a coulomb law of force up to a distance 3.2×10^{-12} cm. It should be pointed out that while the variation of scattering with velocity of the α particle appeared to be normal within the experimental error, yet, owing to the difficulty of estimating the weight of the uranium film with accuracy, the actual number of scattered α particles could not be compared with calculation with any certainty.

The α particle expelled from uranium I has the smallest initial energy known, corresponding to 6.77×10^{-6} erg, or 4.25×10^6 electron-volts. Since the α particle must gain a part at least of its energy in escaping in the repulsive field, it is easy to calculate on a coulomb law of force that, even if the α particle has no initial velocity when it leaves the nuclear structure, it cannot originate from a point less than 7×10^{-12} cm. from the centre of the nucleus of charge 90.

While it is impossible that positively charged particles like the proton or α particle can remain in equilibrium under a coulomb law of repulsive force, the case is quite different if the particles are electrically neutral. A neutral particle can be held in equilibrium by the attractive forces either due to the polarization of the neutral particle by the electric field from the charged central nucleus, or due to magnetic forces arising from the nucleus or to a combination of both types of forces. Preliminary calculations on reasonable assumptions show that the attractive forces due to these causes are of the right order of magnitude to hold the particles in equilibrium when in circulation round the central nucleus.

We thus arrive at a general conception of nuclear structure in which the central charged nucleus is surrounded by a number of uncharged particles. In a Paper before the Franklin Institute in 1924 I put forward a suggestion that the central nucleus was a closely ordered arrangement of α particles and electrons in a semi-crystalline formation, and showed that certain simple arrangements were in fair accord with the charge and mass of some of the atoms. Whatever view we may take on this question, I am inclined to believe that the central nucleus of the heavier elements is a very compact structure, occupying a very small volume of radius of the order 1×10^{-12} cm. The neutral satellites circulating round this nucleus may extend to a distance large compared with the linear dimensions of the main nucleus.

If we take the uncharged α particle as a type of a neutral satellite, it must consist of the helium nucleus which has gained two electrons. These electrons cannot occupy the same positions as in the ordinary helium atom in the free state, for in such a case they would at once be torn off by the intense electric fields due to the nucleus. They are probably much more closely bound to the nucleus, circulating in orbits which are only rendered possible by the distortion of the nuclear structure of the α particle by the intense electrical or magnetic fields from the central nucleus. Such a view seems not unreasonable, for undoubtedly all composite nuclei must suffer serious distortion under the enormous fields which are present in nuclear structures. In fields below a certain critical value, these electron orbits cannot exist, and consequently a neutral α particle in escaping from the nuclear structure would be robbed of its two electrons when the critical field is reached.

We can thus form the following picture of the emission of an α particle from a radioactive element. Occasionally one of the neutral α particles which are probably circulating in quantized orbits is for some cause displaced from its position of equilibrium and has sufficient energy of motion to escape from the attractive field of the central nucleus. When the field falls below the critical value, the neutralizing electrons are removed and fall back towards the nucleus. The α particle, which has now two positive charges, gains additional energy in passing through the repulsive electric field of the nucleus and emerges as a high-speed α particle. It is of interest to note that Fräulein Meitner from consideration of the successive modes of transformation of radioactive atoms concluded that some of the α particles in the nucleus must exist in the neutral state. On the present views all the α particles which escape from radioactive nuclei have their origin as neutral satellites.

We must now follow for a moment the fate of the electrons which are liberated from the neutral α particle. From the known change of charge in a radioactive transformation, it is clear that they must fall back towards the main nucleus, probably describing orbits under the complicated system of forces which must exist close to the nucleus in a region where the forces due to distortion may be all-important. Occasionally one of these electrons is given sufficient energy to escape entirely from the nucleus thus giving rise to a β ray transformation.

In our picture of the nucleus, we thus have a concentrated inner nucleus carrying a positive charge surrounded at a short distance by a number of electrons, and then at a distance a number of neutral satellites circulating round the system. I hope in a later Paper to give a fuller discussion of this type of nuclear structure which offers certain possibilities of interpretation of the wealth of radioactive data at our disposal.

This picture of a nucleus need not be confined only to the radioactive atoms, but is equally applicable to the ordinary atoms. We have so far only discussed the possibility of neutral satellites in the form of α particles of mass 4. It is quite possible, however, that other types of neutral satellites may be present of mass 2 or 3. The possibility of the existence of such nuclei has been drawn attention to by several writers as types of secondary units which play a part in the building-up of nuclear structures, but they have usually been supposed to exist as charged rather than neutral masses. Such secondary neutral units may be able to exist only in the powerful nuclear fields and thus would never be observed in the free state.

This view of nuclear structure at once offers a reasonable explanation of the existence of a number of isotopes of an element of given atomic number. When once the central nucleus is formed, a number of neutral satellites can be added which are kept in equilibrium by attractive forces. Aston has shown in some cases that a large number of isotopes can exist, thus indicating that a number of satellites may be added without disturbing the equilibrium of the nuclear system. He has drawn attention to the remarkable fact that in all cases the odd numbered heavier elements have either no isotopes or two isotopes differing in mass by two units, while even-numbered elements may have a whole group of isotopes. This striking distinction between the elements is paralleled by the observation that odd numbered elements which are disintegrated by the bombardment of α particles emit protons at a much higher average speed than the even-numbered elements. Harkins has also shown that even-numbered elements are much more abundant in nature than the odd-numbered elements.

This difference in isotopic properties between even and odd elements seems very fundamental in character. A possible interpretation may be given on the general view of nuclear structure that we have outlined. It is supposed that the central nucleus is ordinarily made up of helium nuclei carrying two charges arranged in an ordered way, and for an even-numbered element there must be present an even number of electrons.

These would tend to arrange themselves in equilibrium so that the resultant magnetic moment of the system is a minimum. If now, by the addition of a proton or removal of an electron, an odd-numbered element is formed, there may be no longer possible that balance between the magnetic moments as in the case of an even-numbered element. The effect of the resultant magnetic field of the nuclear system may make it impossible for neutral satellites to circulate in permanently stable orbits round the nucleus. If this view be correct, we should anticipate that the nuclear magnetic moment of odd elements should differ markedly from that of even elements. As far as I am aware, there is at present no definite evidence bearing on this point.

In this discussion of atomic constitution, we have been referring for the most part to the heavier elements where the nucleus is considered to be composed mainly of α particles and satellites and where it is to be anticipated that the whole number rule of atomic mass should hold within small limits. If the presence of neutral satellites in the nuclear system depends mainly on the electric charge of the nucleus, the addition of such satellites may be possible only when the nuclear charge exceeds a certain value. For this reason, the constitution of the lighter elements may differ markedly in general features from the heavier elements and the departures from the whole number rule of atomic mass may be more emphasized than for the heavier elements. An accurate determination of the atomic mass of the lighter elements such as Dr. Aston is undertaking is thus of great theoretical, as well as practical importance, as it may give us valuable information on the closeness of the binding of the component protons and electrons.

On account of the uncertainty of the constitution of the lighter elements, it is difficult to connect the disintegration of such elements by α particle bombardment with any special feature of their structure. The views given here are admittedly very speculative in character, but they may serve a useful purpose in suggesting possible lines of attack on this fundamental problem. Although the nucleus of a heavy atom is no doubt very complicated in structure, yet it may present certain simple general features which may be absent or difficult to detect in the lighter elements. For this reason, I am hopeful that we may yet be able by the study of radioactive data to throw light on some of the outstanding features of nuclear constitution of heavy elements.

DEMONSTRATION OF THE ASTROLABE AND SOME OTHER MEDIÆVAL
NAVIGATIONAL AND SURVEYING INSTRUMENTS, ILLUSTRATED
BY LANTERN SLIDES AND EXHIBITS.

By ALLAN FERGUSON, M.A., D.Sc.

OF the two co-ordinates which determine a navigator's position, latitude may be obtained from the height of the pole, or by measuring the meridian zenith distance of a body of known declination (usually the sun), and adding zenith distance and declination together. In the absence of clocks, longitude could be measured only by keeping "a perfect account and reckoning of the way of the shippe."

Among the exhibits shown was a model of the cross-staff. This instrument, which was invented by Regiomontanus in the fifteenth century, comprises a rod with a backsight at one end, and a shorter transverse rod with fore-sights at each of its extremities. The shorter rod slides on the longer, which is graduated so as to give the angle subtended at the eye by two objects simultaneously sighted, such as two stars or the sun and horizon. In Tycho Brahe's form of the instrument the distance (a) between the foresights was adjustable, as well as the distance (b) of the transversal from the back-sight. Tycho reduced his readings to a value x by means of the proportion $a/b = x/1,200$, and looked up the angle corresponding to the value x in the tables of Gemma Frisius, which were, in effect, tables of tangents on a duodecimal basis.

The back staff, which obviated the necessity of looking directly at the sun, was also illustrated.

The astrolabe is made up of three parts. The *tablet* is a stereographic projection on to the plane of the equator from the S. pole of the azimuth and zenith distance circles corresponding to a point in a given latitude. The *rete*, which is pivoted above the tablet, and turns round the celestial pole, is a projection of the celestial sphere, and contains the projections of a few well-known stars and of the ecliptic, the circle of the ecliptic being marked with the sun's position on the celestial sphere for every day of the year. In metal astrolabes the rete is cut in an openwork pattern, on which the stars appear as points at the apices of metal spurs. The *alidade* is a sighting rule provided with pinhole sights by means of which the zenith distance of any celestial body may be measured.

The central problem of the astrolabe is to set the rete so as to show the celestial bodies at any given instant with their correct altitudes and azimuths. This is done by measuring the zenith distance of any known body and turning the rete till that body is over the appropriate zenith distance circle. The rete now shows the aspect of the heavens at the instant of observation, and this aspect includes the position of the sun (whether above or below the horizon). A 24-hour clock face is marked on the edge of the tablet, and the angle which the sun must turn through before reaching the meridian therefore gives the local time. Moreover, inspection of the azimuth circles on the tablet gives, at the instant of observation, the true bearing of every

body marked on the rete. Hence a simultaneous compass bearing of any one of these bodies affords an easy means of correcting a compass. We are, in fact, dealing with three quantities—altitude, azimuth and time—and the astrolabe provides a means of solving problems in which one of these quantities being given, the others are required.

The astrolabe was an essential part of the equipment of the mediæval navigator who, *robur et aes triplex*, adventured forth equipped with astrolabe, cross-staff, compass, a table of the sun's declination, and possibly a chart or two of doubtful value.

Chaucer's treatise on the astrolabe (*Bred and Mylke for Childeren*), written in 1391 for his ten-year-old son Lewis, still remains the standard English work.

Among the instruments shown was a Persian astrolabe, the property of Mr. R. S. Whipple, provided with interchangeable tablets for use at different places, including Baghdad and Teheran.

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CONTENTS.

	PAGE
XX. Acoustical Experiments with a Mechanical Vibrator. By E. MALLETT, D.Sc., M.I.E.E., A.M.I.C.E.	251
XXI. On the Stationary-Wave Method of Measuring Sound-Absorption at Normal Incidence. By E. T. PARIS, D.Sc., F.Inst.P.	269
XXII. A Ball and Tube Flowmeter Suitable for Pressure Circuits. By J. H. AWBERY, B.A., B.Sc., and EZER GRIFFITHS, F.R.S.	296
XXIII. A Gas Analysis Instrument Based on Sound Velocity Measurement. By EZER GRIFFITHS, D.Sc., F.R.S.	300
XXIV. The Scattering of X-Rays and the "J" Phenomenon. By B. L. WORSNOP, B.Sc.	305
XXV. The Characteristics of Thermionic Rectifiers. By Prof. C. L. FORTESCUE	313
XXVI. The Theory of Luminescence in Radioactive Luminous Compound. By J. W. T. WALSH, M.A., M.Sc., F.Inst.P.	318
XXVII. Distortion of Resonance Curves of Electrically-Driven Tuning Forks. By E. MALLETT, D.Sc., M.I.E.E., A.M.I.C.E.	334
XXVIII. The Twelfth Guthrie Lecture: Atomic Nuclei and their Transformations. By Sir ERNEST RUTHERFORD, O.M., P.R.S.	359
Demonstration of the Astrolabe and Some Other Mediæval Navigational and Surveying Instruments, Illustrated by Lantern Slides and Exhibits. By ALLAN FERGUSON, M.A., D.Sc.	373